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EJERCICIOS DE INTEGRAL INDEFINIDA

a) Reducibles a inmediatas.-

$\mathbf{I_1} = \int \operatorname{tg} x dx$; $\mathbf{I_2} = \int \frac{x}{\sqrt{1-x^4}} dx$; $\mathbf{I_3} = \int \frac{dx}{9x^2+4}$; $\mathbf{I_4} = \int \frac{\sqrt{1+x}}{\sqrt{1-x}} dx$; $\mathbf{I_5} = \int \operatorname{tg}^2 x dx$;

$\mathbf{I_6} = \int \frac{\operatorname{sen} 3x}{\cos^2 3x} dx$; $\mathbf{I_7} = \int \frac{x dx}{\sqrt{a^4 - x^4}}$; $\mathbf{I_8} = \int \frac{dx}{\operatorname{sen}^2 x \cdot \cos^2 x}$; $\mathbf{I_9} = \int \operatorname{sen} \frac{5x}{2} \cdot \cos \frac{x}{2} dx$;

$\mathbf{I_{10}} = \int \frac{\sqrt{3} + \operatorname{tg} x}{1 - \sqrt{3} \operatorname{tg} x} dx$; $\mathbf{I_{11}} = \int \frac{x}{(x^2+1)^2} dx$

b) Por sustitución.-

$\mathbf{I_{12}} = \int \frac{x^2}{\sqrt[3]{1+2x}} dx$; $\mathbf{I_{13}} = \int \left(x - \frac{1}{\sqrt{x}} \right)^5 dx$; $\mathbf{I_{14}} = \int \frac{\sqrt[3]{x}}{\sqrt{x} + \sqrt[3]{x}} dx$; $\mathbf{I_{15}} = \int \frac{x^3}{\sqrt{(1-x^2)^3}} dx$ en el intervalo $0 < x < 1$; $\mathbf{I_{16}} = \int x^3 \cdot \sqrt{1+x} \cdot dx$; $\mathbf{I_{17}} = \int \frac{\sqrt{x} - 1}{6(\sqrt[3]{x} + 1)} dx$; $\mathbf{I_{18}} = \int \frac{\sqrt{x} + 1}{2x\sqrt{x}} dx$;

$\mathbf{I_{19}} = \int \frac{x^3 \sqrt{1+x^4}}{\sqrt{1+x^4} + 1} dx$; $\mathbf{I_{20}} = \int \frac{x + \sqrt{2x+1}}{1 + 2\sqrt{2x+1}} dx$; $\mathbf{I_{21}} = \int \frac{x}{2 + \sqrt{1+x}} dx$; $\mathbf{I_{22}} = \int \frac{(e^x - 2)e^x}{e^x + 1} dx$;

$\mathbf{I_{23}} = \int \frac{dx}{x \cdot \ln x}$; $\mathbf{I_{24}} = \int \frac{dx}{x(\ln x + 1)}$; $\mathbf{I_{25}} = \int \frac{dx}{\sqrt{e^x}}$

c) Por partes.-

$\mathbf{I_{26}} = \int \frac{x}{\sqrt{1-x^2}} \cdot \operatorname{arcsen} x \cdot dx$; $\mathbf{I_{27}} = \int \operatorname{arctg} x dx$; $\mathbf{I_{28}} = \int x^3 \cdot e^{-x} \cdot dx$;

$\mathbf{I_{29}} = \int x^2 \operatorname{sen} x \cdot dx$; $\mathbf{I_{30}} = \int x \cdot \operatorname{arctg} x \cdot dx$; $\mathbf{I_{31}} = \int \ln(\operatorname{sen} x) \cdot \cos x \cdot dx$; $\mathbf{I_{32}} = \int \operatorname{sen} x \cdot e^x \cdot dx$;

$\mathbf{I_{33}} = \int x \cdot \operatorname{sen} x \cdot dx$; $\mathbf{I_{34}} = \int \frac{x}{\cos^2 x} dx$; $\mathbf{I_{35}} = \int \frac{x}{\operatorname{sen}^2 x} dx$; $\mathbf{I_{36}} = \int e^{2x} \operatorname{sen} 3x \cdot dx$

d) Funciones racionales.-

$\mathbf{I_{37}} = \int \frac{x^3 + 2x + 1}{x(x+1)} dx$; $\mathbf{I_{38}} = \int \frac{x^3 + 2x + 1}{x^2 + x + 1} dx$; $\mathbf{I_{39}} = \int \frac{dx}{x^4 - 1}$; $\mathbf{I_{40}} = \int \frac{2x+3}{(x-2)(x+5)} dx$;

$\mathbf{I_{41}} = \int \frac{x}{(x+1)(x+2)(x+3)} dx$; $\mathbf{I_{42}} = \int \frac{x}{x^3 - 3x + 2} dx$; $\mathbf{I_{43}} = \int \frac{dx}{x^3 + 1}$; $\mathbf{I_{44}} = \int \frac{dx}{x^3 - 1}$;

$\mathbf{I_{45}} = \int \frac{x^2 + 1}{(x+1)^2 (x-1)} dx$; $\mathbf{I_{46}} = \int \frac{x \cdot \ln x}{(x^2+1)^2} dx$ *(previamente por partes)* ;

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$I_{47} = \int \frac{3x^2+x+2}{x^3-x^2-x+1}dx$ y hallar la primitiva que pasa por el punto P(0,5);

$I_{48} = \int \frac{5x+3}{2x^2-4x+10}dx$

e) Funciones trigonométricas.-

$I_{49} = \int \frac{1}{\operatorname{sen} x \cdot \cos^2 x}dx$; $I_{50} = \int \frac{\operatorname{sen}^3 x}{1+\cos^2 x}dx$ para $0 < x < \pi$; $I_{51} = \int \frac{\cos^3 x}{4\operatorname{sen}^2 x - 1}dx$ en el intervalo $-\frac{\pi}{2} < x < \frac{\pi}{2}$; $I_{52} = \int \frac{dx}{\operatorname{sen}^2 x - 4\operatorname{sen} x \cos x + 5\cos^2 x}$ en el intervalo $-\frac{\pi}{2} < x < \frac{\pi}{2}$;

$I_{53} = \int \frac{1}{1+\operatorname{sen} x}dx$; $I_{54} = \int \frac{1}{1+2\cos x}dx$ $I_{55} = \int \frac{1}{\operatorname{sen} x + \cos x}dx$; $I_{56} = \int \frac{1}{\operatorname{sen} x + \cos x + 1}dx$;

$I_{57} = \int \operatorname{sen}^2 x \cdot dx$; $I_{58} = \int \operatorname{sen}^3 x \cdot dx$; $I_{59} = \int \operatorname{sen}^4 x \cdot dx$

f) Funciones irracionales.-

$I_{60} = \int \frac{dx}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}$; $I_{61} = \int \frac{dx}{\sqrt{x^2+6x+5}}$; $I_{62} = \int \frac{dx}{\sqrt{-x^2+4x-3}}$;

$I_{63} = \int \frac{dx}{\sqrt{x^2-3x+2}}$; $I_{64} = \int \frac{dx}{\sqrt{x^2+1}}$; $I_{65} = \int \frac{dx}{\sqrt{x-x^2}}$

SOLUCIONES

$I_1 = -\ln(\cos x) + C$; $I_2 = \frac{1}{2}\text{arc sen } x^2 + C$; $I_3 = \frac{1}{6}\text{arctg}\frac{3x}{2} + C$; $I_4 =$ *(multiplicando numerador y denominador por $\sqrt{1+x}$)* $= \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx =$ *(son inmediatas)* $=$

$= \text{arc sen } x - \sqrt{1-x^2} + C$.

$I_5 = \int \frac{\operatorname{sen}^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx = \int \frac{1}{\cos^2 x}dx - \int \frac{\cos^2 x}{\cos^2 x}dx = \operatorname{tg} x - x + C$;

$I_6 = \int \operatorname{sen} 3x \cdot \cos^{-2} 3x \cdot dx =$ *(inmediata)* $= -\frac{1}{3}\cdot\frac{\cos^{-1} 3x}{-1} + C = \frac{1}{3\cos 3x} + C$; $I_7 = \frac{1}{2}\text{arcsen}\frac{x^2}{a^2} + C$;

$I_8 = \int \frac{4}{4\cdot\operatorname{sen}^2 x\cdot\cos^2 x}dx = -2\int \frac{-2}{\operatorname{sen}^2 2x}dx =$ *(inmediata)* $= -2\operatorname{cotg} 2x + C$

$I_9 =$ *(usaremos la fórmula trigonométrica **senp·cosq** = $= \frac{1}{2}\left[\operatorname{sen}(p+q) + \operatorname{sen}(p-q)\right]$)* $=$

$= \frac{1}{2}\int\left(\operatorname{sen} 3x + \operatorname{sen} 2x\right)dx =$ *(inmediata)* $= -\frac{1}{6}\cos 3x - \frac{1}{4}\cos 2x + C$;

$I_{10} =$ *(poniendo tg x = $\frac{senx}{cosx}$ y simplificando)* $= \int \frac{\sqrt{3}\cos x + \operatorname{sen} x}{\cos x - \sqrt{3}\operatorname{sen} x}dx =$

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= *(inmediata)* = $-\ln\left|\cos x - \sqrt{3}\,\text{sen}\ x\right| + C$; **I$_{11}$** $= -\dfrac{1}{2\left(x^2+1\right)} + C$;

I$_{12}$ $= \dfrac{3}{320}\sqrt[3]{(2x+1)^2}\left(20x^2-12x+9\right)+C$; **I$_{13}$** $= 5\ln|x| + \dfrac{x^6}{6} + \dfrac{10x^{9/2}}{9} + \dfrac{10x^3}{3} + + \dfrac{20x^{3/2}}{3} -$

$-\dfrac{2x^{-3/2}}{3} + C$; **I$_{14}$** $= -6\ln(x^{1/6}+1) + \dfrac{6}{5}x^{5/6} - \dfrac{3}{2}x^{2/3} + 2x^{1/2} - 3x^{1/3} + 6x^{1/6} + C$;

I$_{15}$ $= \dfrac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} + C$; **I$_{16}$** $= \dfrac{2}{315}(x+1)^{3/2}(35x^3 - 30x^2 + 24x - 16) + C$;

I$_{17}$ $= -\sqrt[6]{x} + \dfrac{\sqrt[3]{x}}{2} + \dfrac{\sqrt{x}}{3} - \dfrac{\sqrt[3]{x^2}}{4} - \dfrac{\sqrt[6]{x^5}}{5} + \dfrac{x\sqrt[6]{x}}{7} + \text{arctg}\sqrt[6]{x} - \dfrac{\ln\left(1+\sqrt[3]{x}\right)}{2} + C$; **I$_{18}$** $= \dfrac{1}{2}\ln x - \dfrac{1}{\sqrt{x}} +$

$+C$; **I$_{19}$** $= \dfrac{\ln\left(\sqrt{x^4+1}+1\right)}{2} - \dfrac{2\sqrt{x^4+1}-1}{4} + C$; **I$_{20}$** $= \dfrac{7}{32}\ln\left|2\sqrt{2x+1}+1\right| +$

$+\dfrac{1}{48}\sqrt{2x+1}(8x-17) + \dfrac{3}{8}x + C$; **I$_{21}$** $= \dfrac{2}{3}\left[(x+10)\sqrt{x+1} - 3x\right] - 12\ln\left|\sqrt{x+1}+2\right| + C$;

I$_{22}$ $= e^x - \ln(e^x+1) + C$; **I$_{23}$** $= \ln|\ln x| + C$; **I$_{24}$** $= \ln|\ln x + 1| + C$; **I$_{25}$** $= \dfrac{-2}{\sqrt{e^x}} + C$;

I$_{26}$ $= x - \sqrt{1-x^2}\ \text{arc sen}\ x + C$; **I$_{27}$** $= x\cdot\text{arc tg}\ x - \dfrac{1}{2}\ln(x^2+1) + C$; **I$_{28}$** $= -e^{-x}(x^3+3x^2+6x+6) +$

$+ C$; **I$_{29}$** $= (2-x^2)\cos x + 2x\cdot\text{sen}\ x + C$; **I$_{30}$** $= \dfrac{1}{2}[(x^2+1)\text{arc tg}\ x - x] + C$;

I$_{31}$ $= \text{sen}\ x\cdot\ln|\text{sen}\ x| - \text{sen}\ x + C$; **I$_{32}$** $= \dfrac{1}{2}e^x(\text{sen}\ x - \cos x) + C$; **I$_{33}$** $= \text{sen}\ x - x\cdot\cos x + C$;

I$_{34}$ $= \ln|\cos x| + x\cdot\text{tg}\ x + C$; **I$_{35}$** $= \ln|\text{sen}\ x| - x\cdot\text{cotg}\ x + C$; **I$_{36}$** $= \dfrac{e^{2x}}{13}(2\text{sen}\ 3x - 3\cos 3x) +$
$+C$; **I$_{37}$** = Efectuamos la división indicada en el integrando y se obtiene:

$$\frac{x^3+2x+1}{x(x+1)} = x - 1 + \frac{3x+1}{x(x+1)}$$

Descomponiendo esta última fracción en suma de fracciones simples:

$$\frac{3x+1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{(A+B)x+A}{x(x+1)} \Rightarrow \begin{cases} A = 1 \\ B = 2 \end{cases}$$

Así pues **I$_{37}$**= $\displaystyle\int\left(x - 1 + \frac{1}{x} + \frac{2}{x+1}\right)dx$ = *(inmediata)* = $\dfrac{1}{2}x^2 - x + \ln|x| + 2\cdot\ln|x+1| + C$;

I$_{38}$ $= -x + \dfrac{1}{2}x^2 + \ln(x^2+x+1) + \dfrac{2\sqrt{3}}{3}\text{arctg}\dfrac{2x+1}{\sqrt{3}} + C$; **I$_{39}$** $= \dfrac{1}{4}\ln\left|\dfrac{x-1}{x+1}\right| - \dfrac{1}{2}\text{arc tg}\ x + C$; **I$_{40}$** =

$\ln|x^2+3 \quad -10| + C$ **I$_{41}$** $= -\dfrac{1}{2}\ln|x+1| + 2\ln|x+2| + \dfrac{3}{2}\ln|x+3| + C$; **I$_{42}$** =

$-\dfrac{1}{3}\cdot\dfrac{1}{x-1} + \dfrac{2}{9}\ln\left|\dfrac{x-1}{x+2}\right| + C$;

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I₄₃ $= \frac{1}{\sqrt{3}}\,\text{arctg}\frac{2x-1}{\sqrt{3}} + \frac{1}{3}\ln|1+x| - \frac{1}{6}\ln\left|x^2 - x + 1\right| + C;$

I₄₄ $= -\frac{1}{\sqrt{3}}\,\text{arctg}\frac{2x+1}{\sqrt{3}} + \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln\left|x^2 + x + 1\right| + C;$ **I₄₅** $= \frac{1}{x+1} + \frac{1}{2}\ln\left|x^2 - 1\right| + C;$

I₄₆ $= \frac{1}{2}\ln|x| - \frac{\ln|x|}{2\left(1+x^2\right)} - \frac{\ln\left(1+x^2\right)}{4} + C;$ **I₄₇** $= \ln(x-1)^2 + \ln(x+1) - \frac{3}{x-1} + C$; para C = 2 pasa por (0,5); **I₄₈** $= \frac{5}{4}\ln\left[1 + \left(\frac{x-1}{2}\right)^2\right] + 2\,\text{arctg}\left(\frac{x-1}{2}\right) + C;$ **I₄₉** = *(impar en seno → cambio **cosx = t**)* =

$\int \frac{1}{t^2\left(t^2-1\right)}dt = \int \frac{1}{t^2(t+1)(t-1)}dt =$ *(descomposición en fracciones simples)* $= \int\left(\frac{-1}{t^2} + \frac{-1/2}{t+1} + \frac{1/2}{t-1}\right)dt =$

= *(inmediata)* = $\frac{1}{t} - \ln\sqrt{|t+1|} + \ln\sqrt{|t-1|} + C$ = *(deshaciendo el cambio y agrupando logaritmos)* =

= $\sec x + \ln\sqrt{\frac{1-\cos x}{1+\cos x}} + C;$ **I₅₀** = *(impar en seno → cambio **cosx = t**)* = $\int \frac{t^2-1}{t^2+1}dx =$

= *(dividiendo)* = $\int\left(1 - \frac{2}{t^2+1}\right)dx =$ *(inmediata)* = t – 2arc tg t + C = *(deshaciendo el cambio)* =

= cos x – 2arc tg (cos x) + C; **I₅₁** = *(impar en coseno → cambio **senx = t**)* = $\int \frac{1-t^2}{4t^2-1}dx =$ *(dividiendo)* =

= $\int\left(\frac{-1}{4} + \frac{3/4}{4t^2-1}\right)dx = \int\left(\frac{-1}{4} + \frac{3/4}{(2t+1)(2t-1)}\right)dx =$ *(descomposición en fracciones simples)* =

= $\int\left(\frac{-1}{4} + \frac{-3/8}{2t+1} + \frac{3/8}{2t-1}\right)dx =$ *(inmediata)* = $\frac{-1}{4}t - \frac{3}{16}\ln|2t+1| + \frac{3}{16}\ln|2t-1| + C =$ *(deshaciendo el cambio y agrupando logaritmos)* = $-\frac{1}{4}\,\text{sen}\,x + \frac{3}{16}\ln\left|\frac{2\,\text{sen}\,x - 1}{2\,\text{sen}\,x + 1}\right| + C;$ **I₅₂** = *(par en seno y coseno, simultáneamente → cambio **tg x = t** →* $\left\{\begin{array}{l} \cos^2 x = \frac{1}{1+t^2} \\ \text{sen}^2 x = \frac{t^2}{1+t^2} \\ dx = \frac{dt}{1+t^2} \end{array}\right.$ *)* = $\int \frac{dt}{t^2 - 4t + 5} = \int \frac{dt}{(t-2)^2 + 1} =$ *(inmediata)* =

= arc tg (t–2) + C = *(deshaciendo el cambio)* = arc tg(tg x – 2) + C;

I₅₃ = *(cambio* $tg\frac{x}{2} = t \rightarrow \left\{\begin{array}{l} senx = \frac{2t}{1+t^2} \\ dx = \frac{2}{1+t^2}dt \end{array}\right.$ *)* = $\int \frac{2}{t^2 + 2t + 1}dt = \int \frac{2}{(t+1)^2}dt =$ *(inmediata)* = $\frac{-2}{t+1} + C =$

= *(deshaciendo el cambio)* = $\frac{-2}{1 + tg\frac{x}{2}}$ +C.; **I₅₄** = *(cambio* $tg\frac{x}{2} = t \rightarrow \left\{\begin{array}{l} cosx = \frac{1-t^2}{1+t^2} \\ dx = \frac{2}{1+t^2}dt \end{array}\right.$ *)* = $\int \frac{2}{3-t^2}dt =$

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$= \displaystyle\int \frac{2}{\left(\sqrt{3}+t\right)\left(\sqrt{3}-t\right)}dt =$ *(descomposición en fracciones simples)* $= \displaystyle\int \left(\frac{\frac{1}{\sqrt{3}}}{\left(\sqrt{3}+t\right)} + \frac{\frac{1}{\sqrt{3}}}{\left(\sqrt{3}-t\right)} \right) dt =$

$=$ *(inmediata)* $= \dfrac{1}{\sqrt{3}}\ln\left|\sqrt{3}+t\right| - \dfrac{1}{\sqrt{3}}\ln\left|\sqrt{3}-t\right| + C =$ *(deshaciendo el cambio y agrupando logaritmos)* $=$

$= \dfrac{1}{\sqrt{3}}\ln\left|\dfrac{\sqrt{3}+tg\frac{x}{2}}{\sqrt{3}-tg\frac{x}{2}}\right| + C$; **I₅₅** $=$ *(cambio* $tg\dfrac{x}{2} = t \rightarrow \begin{cases} senx = \dfrac{2t}{1+t^2} \\ cosx = \dfrac{1-t^2}{1+t^2} \\ dx = \dfrac{2}{1+t^2}dt \end{cases}$ *)* $= \displaystyle\int \frac{2}{2t+1-t^2}dt =$

$= \displaystyle\int \frac{-2}{\left(t-1-\sqrt{2}\right)\left(t-1+\sqrt{2}\right)}dt =$ *(descomposición en fracciones simples)* $= \displaystyle\int \frac{-2}{\left(t-1-\sqrt{2}\right)\left(t-1+\sqrt{2}\right)}dt =$

$= \displaystyle\int \left(\frac{-\frac{1}{\sqrt{2}}}{\left(t-1-\sqrt{2}\right)} + \frac{\frac{1}{\sqrt{2}}}{\left(t-1+\sqrt{2}\right)} \right) dt =$ *(inmediata)* $= -\dfrac{1}{\sqrt{2}}\ln\left|t-1-\sqrt{2}\right| + \dfrac{1}{\sqrt{2}}\ln\left|t-1+\sqrt{2}\right| + C =$

$=$ *(deshaciendo el cambio y agrupando logaritmos)* $= \dfrac{1}{\sqrt{2}}\ln\left|\dfrac{tg\frac{x}{2}-1+\sqrt{2}}{tg\frac{x}{2}-1-\sqrt{2}}\right| + C$;

I₅₆ $=$ *(cambio* $tg\dfrac{x}{2} = t \rightarrow \begin{cases} senx = \dfrac{2t}{1+t^2} \\ cosx = \dfrac{1-t^2}{1+t^2} \\ dx = \dfrac{2}{1+t^2}dt \end{cases}$ *)* $= \displaystyle\int \frac{2}{2t+1-t^2+1+t^2}dt =$

$= \displaystyle\int \frac{1}{t+1}dt =$ *(inmediata)* $= \ln|t+1| + C =$ *(deshaciendo el cambio)* $= \ln\left|1+tg\dfrac{x}{2}\right| + C$;

I₅₇ = (usaremos la fórmula trigonométrica $sen^2x = \dfrac{1-cos2x}{2}$) $= \dfrac{1}{2}\displaystyle\int\left(1-\cos 2x\right)\cdot dx =$

$=$ (inmediata) $= \dfrac{1}{2}\left(x - \dfrac{sen2x}{2}\right) + C$; **I₅₈ =**(impar en seno → cambio **cosx = t**) $= \displaystyle\int\left(t^2-1\right)dt =$

(inmediata) $= \dfrac{t^3}{3} - t + C =$ (deshaciendo el cambio) $= \dfrac{\cos^3 x}{3} - \cos x + C$; **I₅₉ =** (usaremos las fórmulas trigonométricas $sen^2x = \dfrac{1-cos2x}{2}$ y $\mathbf{cos^2x} = \dfrac{1+cos2x}{2}$)$= \displaystyle\int\left(\frac{1-\cos 2x}{2}\right)^2 dx =$

$\dfrac{1}{4}\displaystyle\int\left(1-2\cos 2x+\cos^2 2x\right)dx =$

$= \dfrac{1}{4}\displaystyle\int\left(1-2\cos 2x+\frac{1+\cos 4x}{2}\right)dx = \dfrac{1}{8}\displaystyle\int\left(3-4\cos 2x+\cos 4x\right)dx =$ (inmediata) $=$

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$\frac{1}{8}\left(3x-2\operatorname{sen}2x+\frac{\operatorname{sen}4x}{4}\right)+C$; **I$_{60}$** = (cambio x = t^6) = $\int\frac{6t^5dt}{(1+t^2)t^3}=\int\frac{6t^2dt}{1+t^2}$ = (dividiendo) = $=\int\left(6-\frac{6}{t^2+1}\right)dt$ = (inmediata) = 6t – 6·arctgt + C = (deshaciendo el cambio) = $6\sqrt[6]{x}-6\operatorname{arctg}\sqrt[6]{x}$ +C; **I$_{61}$** = (cambio $\sqrt{x^2+6x+5}$ = x + t → x = $\frac{t^2-5}{6-2t}$; dx = $\frac{-2t^2+12t-10}{(6-2t)^2}dt$, sustituyendo y simplificando) = $\int\frac{1}{3-t}dt$ = (inmediata) = ln|3 – t|+C = (deshaciendo el cambio) =

= $-\ln\left|3+x-\sqrt{x^2+6x+5}\right|$+C; **I$_{62}$** = $\int\frac{dx}{\sqrt{1-(x-2)^2}}$ = (inmediata) = arc sen (x – 2) + C ó también **I$_{62}$** = –2arc tg $\sqrt{\frac{x-3}{1-x}}$+C; **I$_{63}$** = (cambio $\sqrt{x^2-3x+2}$ = x + t → x = $\frac{2-t^2}{2t+3}$; dx = $\frac{-2t^2-6t-4}{(2t+3)^2}dt$, sustituyendo y simplificando) = $\int\frac{-2}{3+2t}dt$ = (inmediata) = –ln|3+2t|+C = (deshaciendo el cambio) =

= $-\ln\left|-2x+3+2\sqrt{x^2-3x+2}\right|$+C; **I$_{64}$** = (cambio $\sqrt{x^2+1}$ = x + t → x = $\frac{1-t^2}{2t}$; dx = $\frac{-t^2-1}{2t^2}dt$, sustituyendo y simplificando) = = $\int\frac{-1}{t}dt$ = (inmediata) = –ln|t|+C = (deshaciendo el cambio)

= $-\ln\left|-x+\sqrt{x^2+1}\right|$+C; **I$_{65}$** = *(cambio* $\sqrt{x-x^2}$ = x t → x = $\frac{1}{1+t^2}$; dx = $\frac{-2t}{(1+t^2)^2}dt$, *sustituyendo y simplificando)* = $\int\frac{-2}{1+t^2}dt$ = *(inmediata)* = –2arctg t+C = *(deshaciendo el cambio)* =

= –2arc tg $\frac{\sqrt{x-x^2}}{x}$ +C.

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MÁS EJERCICIOS CON SOLUCIÓN:

Racionales:

1.- $\int\frac{dx}{x^2-9}=\frac{1}{6}\ln|x-3|-\frac{1}{6}\ln|x+3|+C$

2.- $\int\frac{xdx}{x^2-3x-4}=\frac{1}{5}\ln|x+1|+\frac{4}{5}\ln|x-4|+C$

3.- $\int\frac{x^4dx}{(1-x)^3}=-\frac{1}{2}x^2-3x-6\ln|1-x|-\frac{4}{1-x}+\frac{1}{2(1-x)^2}+C$

4.- $\int\frac{3x-2}{x^2+x+1}dx=\frac{3}{2}\ln(x^2+x+1)-\frac{7}{\sqrt{3}}artg\frac{2x+1}{\sqrt{3}}+C$

5.- $\int\frac{3x+5}{x^3-x^2-x+1}dx=\frac{1}{2}\ln|x+1|-\frac{1}{2}\ln|x-1|-\frac{4}{x-1}+C$

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6.- $\int \frac{dx}{x(x+1)^2} = \frac{1}{1+x} + \ln|x| - \ln|x+1| + C$

7.- $\int \frac{x^3+x+1}{x(x^2+1)}dx = x + \ln|x| - \frac{1}{2}\ln(x^2+1) + C$

8.- $\int \frac{x^2-5x+9}{x^2-5x+6}dx = x + 3\ln|x-3| - 3\ln|x-2| + C$

9.- $\int \frac{x^2+3x-4}{x^2-2x-8}dx = x + 4\ln|x-4| + \ln|x+2| + C$

10.- $\int \frac{x^4-x^3-x-1}{x^3-x^2}dx = \frac{x^2}{2} + 2\ln|x| - \frac{1}{x} - 2\ln|x-1| + C$

11.- $\int \frac{dx}{(x^2-1)^2} = -\frac{1}{4}\ln|x-1| - \frac{1}{4(x-1)} + \frac{1}{4}\ln|x+1| - \frac{1}{4(x+1)} + C$

12.- $\int \frac{dx}{(x^2+1)(x+1)} = \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln\left(x^2+1\right) + \frac{1}{2} artgx + C$

13.- $\int \frac{x^4}{x^4-1}dx = x + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2} artgx + C$

14.- $\int \frac{x-5}{x^2-2x+2}dx = \frac{1}{2}\ln(x^2-2x+2) - 4artg(x-1) + C$

15.- $\int \frac{dx}{(x+2)^2(x+3)^2} = 2\ln\left|\frac{x+3}{x+2}\right| - \frac{1}{x+2} - \frac{1}{x+3} + C$

16.- $\int \frac{x^3+2x+1}{x(x+1)}dx = -x + \frac{1}{2}x^2 + \ln|x| + 2\ln|x+1| + C$

17.- $\int \frac{x^3+2x+1}{x^2+x+1}dx = -x + \frac{1}{2}x^2 + \ln(x^2+x+1) + C$

18.- $\int \frac{dx}{x^3+1} = \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{\sqrt{3}} artg\,\frac{2x-1}{\sqrt{3}} + C$

Funciones trigonométricas:

1.- $\int \cos^3 x dx = \operatorname{sen}x - \frac{1}{3}\operatorname{sen}^3 x + C$

2.- $\int \operatorname{sen}^2 x \cos^3 x dx = \frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5} + C$

3.- $\int \frac{\cos^5 x}{\operatorname{sen}^3 x} dx = \frac{\operatorname{sen}^2 x}{2} - \frac{1}{2\operatorname{sen}^2 x} - 2\ln|\operatorname{sen}x| + C$

4.- $\int sen3x\cos 5x dx = \frac{-\cos 8x}{16} + \frac{\cos 2x}{4} + C$

5.- $\int sen\frac{x}{3}\cos\frac{2x}{3}dx = \frac{3}{2}\cos\frac{x}{3} - \frac{\cos x}{2} + C$

6.- $\int \frac{dx}{3+5\cos x} = \frac{1}{4}\ln\left|2 + tg\frac{x}{2}\right| - \frac{1}{4}\ln\left|2 - tg\frac{x}{2}\right| + C$

7.- $\int \frac{\cos x}{1+\cos x}dx = x - tg\frac{x}{2} + C \qquad tg\frac{x}{2} = t$

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8.- $\int \frac{\cos^2 x}{sen^4 x} dx = -\frac{1}{3tg^3 x} + C \qquad tgx = t$

9.- $\int \frac{dx}{sen^2 x - 5 senx \cos x} = \frac{1}{5}\ln|tgx - 5| - \frac{1}{5}\ln|tgx| + C$

10.- $\int \frac{1+tgx}{1-tgx} dx = -\ln|\cos x - senx| + C$

11.- $\int \frac{dx}{8 - 4senx + 7\cos x} = \ln\left|tg\frac{x}{2} - 5\right| - \ln\left|tg\frac{x}{2} - 3\right| + C$

12.- $\int sen^2 x \cos^2 x dx = \frac{x}{8} - \frac{sen4x}{32} + C$

13.- $\int \frac{sen^3 x}{1+\cos^2 x} dx = \cos x - 2artg(\cos x) + C$

14.- $\int \frac{dx}{sen^2 x - 4senx\cos x + 5\cos^2 x} = artg(tgx - 2) + C$

15.- $\int \frac{1}{1+senx} dx = \frac{-2}{1+tg\frac{x}{2}} + C$

16.- $\int sen^4 x dx = \frac{1}{2}\left(x - \frac{sen2x}{2}\right) - \frac{1}{16}\left(2x - \frac{sen4x}{2}\right) + C$

17.- $\int \frac{1}{senx + \cos x + 1} dx = \ln\left|1 + tg\frac{x}{2}\right| + C$

Irracionales:

1.- $\int \frac{1+x}{1+\sqrt{x}} dx = \frac{2}{3}\sqrt{x^3} - x + 4\sqrt{x} - 4\ln|1+\sqrt{x}|$

2.- $\int \frac{dx}{x\sqrt{2x+1}} = \ln\left|\sqrt{2x+1} - 1\right| + \ln\left|\sqrt{2x+1} + 1\right| - 2\ln\sqrt{2x+1} + C$

3.- $\int \frac{dx}{\left(1+\sqrt[3]{x}\right)\sqrt{x}} = 6\sqrt[6]{x} - 6artg\sqrt[6]{x} + C$

4.- $\int \frac{dx}{\sqrt{x^2+6x+5}} = -\ln\left|3 + x - \sqrt{x^2+6x+5} +\right| + C$

5.- $\int \frac{dx}{\sqrt{-x^2+4x-3}} = -2artg\sqrt{\frac{x-3}{1-x}} + C$

6.- $\int \frac{dx}{\sqrt{x^2-3x+2}} = \ln\left|2x - 3 + 2\sqrt{x^2-3x+2}\right| + C$

7.- $\int \frac{dx}{\sqrt{x-x^2}} = -2artg\frac{\sqrt{x-x^2}}{x} + C$

8.- $\int \sqrt{3-x^2} dx = \frac{3}{2}\left[arsen\frac{x}{\sqrt{3}} + \frac{x}{3}\sqrt{3-x^2}\right] + C$

9.- $\int \sqrt{4-9x^2} dx = \frac{4}{3}\left[arsen\frac{3x}{2} + \frac{3x}{2}\sqrt{4-9x^2}\right] + C$

10.- $\int \frac{dx}{x^2\sqrt{x^2+1}} = -\frac{1}{sen(artgx)} + C$