Diseño de experimentos

Estadística 2002-2003

Bloques aleatorizados

		Fluorita				
		0%	1%	2%	3%	4%
M	1	15.02	11.86	9.94	12.45	13.23
e	2	8.42	10.15	8.54	6.98	8.93
z	3	18.31	16.84	15.86	14.64	15.96
c	4	10.49	10.52	8.04	10.50	10.34
l	5	9.78	9.59	6.96	8.15	9.24
a	6	9.28	8.84	7.04	6.66	9.46

Se desea estudiar el efecto de la Fluorita en la reducción del coste energético en la fabricación de cemento. Se emplean 6 mezclas distintas de materias primas.

Modelo

Tratamientos

Bloques

	1	2	$\cdots$	I
1	y_{11}	y_{21}	$\cdots$	y_{I1}
2	y_{12}	y_{22}	$\cdots$	y_{I2}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
J	y_{1J}	y_{2J}	$\cdots$	y_{IJ}

$$y_{ij} = \mu + \alpha_i + \beta_j + u_{ij}$$

- Normalidad
- Independencia
- Homocedasticidad

μ : Media global

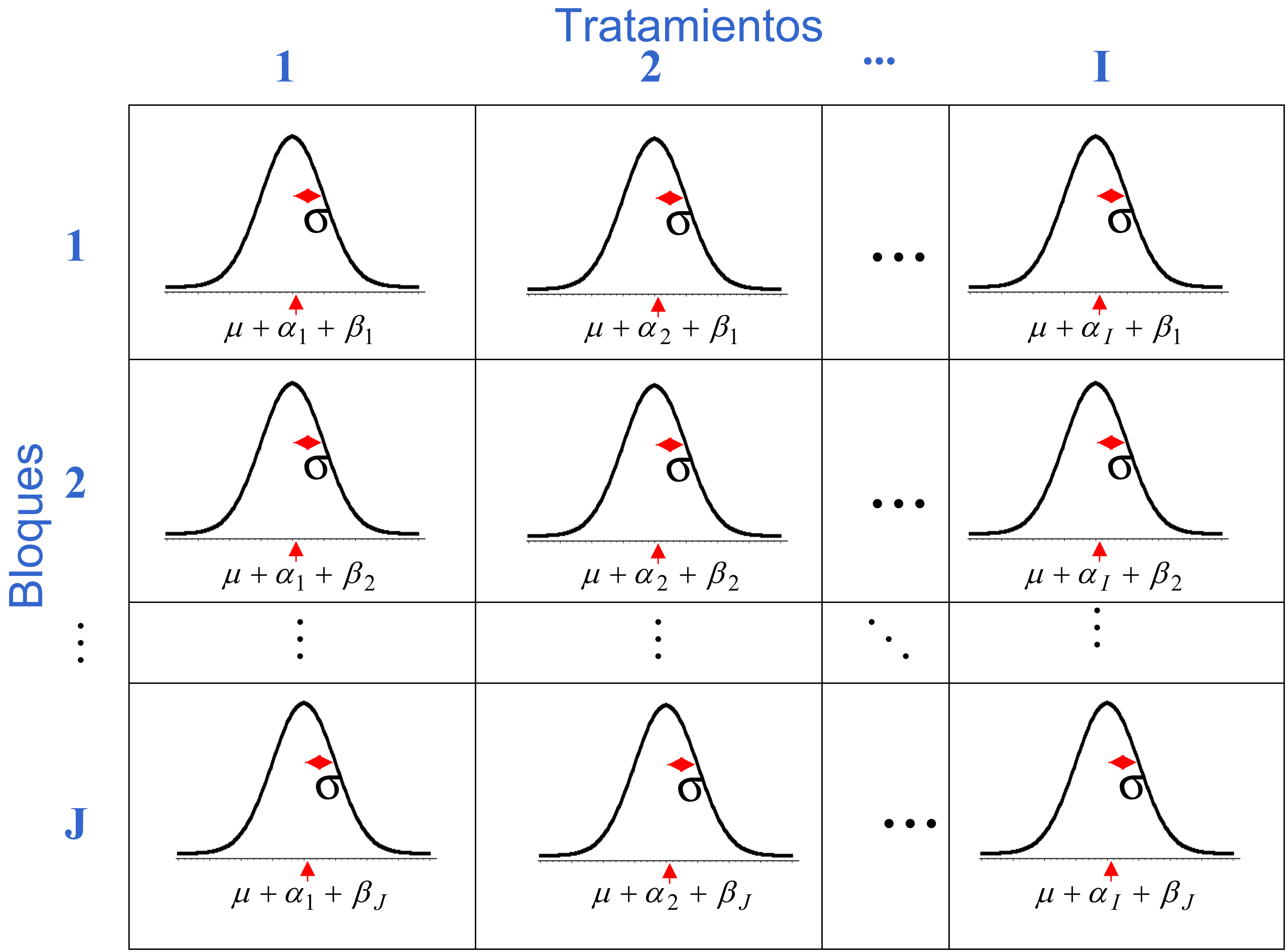
α_i : Efecto del tratamiento i, i=1,...,I

β_j : Efecto del bloque j, j=1,2,...,J

u_{ij} : Componente aleatoria $N(0,\sigma^2)$

$\sum_{i=1}^{I} \alpha_i = 0$

$\sum_{j=1}^{J} \beta_j = 0$



Tratamientos
1
2
...
I
Bloques
1
2
J
σ
$\mu + \alpha_1 + \beta_1$
$\mu + \alpha_2 + \beta_1$
$\mu + \alpha_I + \beta_1$
$\mu + \alpha_1 + \beta_2$
$\mu + \alpha_2 + \beta_2$
$\mu + \alpha_I + \beta_2$
$\mu + \alpha_1 + \beta_J$
$\mu + \alpha_2 + \beta_J$
$\mu + \alpha_I + \beta_J$

Estimación del modelo

$$\text{Parámetros}:\begin{cases}\mu :\to 1\\ \alpha_{\mathrm{i}} :\to I-1\\ \beta_{\mathrm{j}} :\to J-1\\ \sigma^2 :\to 1\end{cases}\qquad \text{Estimadores}:\begin{cases}\hat{\mu}=\bar{y}_{\bullet\bullet}\\ \hat{\alpha}_i=\bar{y}_{i\bullet}-\bar{y}_{\bullet\bullet}\\ \hat{\beta}_j=\bar{y}_{\bullet j}-\bar{y}_{\bullet\bullet}\\ \hat{\sigma}^2=\hat{s}_R^2=\dfrac{\sum\sum e_{ij}^2}{(I-1)(J-1)}\end{cases}$$

$$\boxed{\bar{y}_{i\bullet}=\frac{\sum_{j=1}^{J} y_{ij}}{J}\qquad \bar{y}_{\bullet j}=\frac{\sum_{i=1}^{I} y_{ij}}{I}\qquad \bar{y}_{\bullet\bullet}=\frac{\sum_{i=1}^{I}\sum_{j=1}^{J} y_{ij}}{n}}$$

$$\boxed{\begin{aligned}y_{ij}&=\mu+\alpha_i+\beta_j+u_{ij}\\ y_{ij}&=\hat{\mu}+\hat{\alpha}_i+\hat{\beta}_j+e_{ij}\end{aligned}}\;\rightarrow\;\begin{aligned}e_{ij}&=y_{ij}-\hat{\mu}-\hat{\alpha}_i-\hat{\beta}_j\\ &=y_{ij}-\bar{y}_{i\bullet}-\bar{y}_{\bullet j}+\bar{y}_{\bullet\bullet}\end{aligned}$$

Estimación

	1	2	$\cdots$	I		$\hat{\beta}_j$
1	y_{11}	y_{21}	$\cdots$	y_{I1}	$\overline{y}_{\bullet 1}$	$\overline{y}_{\bullet 1} - \overline{y}_{\bullet\bullet}$
2	y_{12}	y_{22}	$\cdots$	y_{I2}	$\overline{y}_{\bullet 2}$	$\overline{y}_{\bullet 2} - \overline{y}_{\bullet\bullet}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
J	y_{1J}	y_{2J}	$\cdots$	y_{IJ}	$\overline{y}_{\bullet J}$	$\overline{y}_{\bullet J} - \overline{y}_{\bullet\bullet}$
	$\overline{y}_{1\bullet}$	$\overline{y}_{2\bullet}$	$\cdots$	$\overline{y}_{I\bullet}$	$\overline{y}_{\bullet\bullet}$	
$\hat{\alpha}_i$	$\overline{y}_{1\bullet} - \overline{y}_{\bullet\bullet}$	$\overline{y}_{2\bullet} - \overline{y}_{\bullet\bullet}$	$\cdots$	$\overline{y}_{I\bullet} - \overline{y}_{\bullet\bullet}$		

Estimación (ejemplo)

		Fluorita						
		0%	1%	2%	3%	4%		
M	1	15.02	11.86	9.94	12.45	13.23	12.50	1.77
e	2	8.42	10.15	8.54	6.98	8.93	8.60	-2.13
z	3	18.31	16.84	15.86	14.64	15.96	16.32	5.59
c	4	10.49	10.52	8.04	10.50	10.34	9.98	-0.76
l	5	9.78	9.59	6.96	8.15	9.24	8.74	-1.99
a	6	9.28	8.84	7.04	6.66	9.46	8.26	-2.48
		11.88	11.30	9.40	9.90	11.19	10.73	
		1.15	0.57	-1.34	-0.84	0.46		

β_j

α_i

Residuos: Varianza residual

$$e_{ij} = y_{ij} - \hat{\mu} - \hat{\alpha}_i - \hat{\beta}_j = y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} + \bar{y}_{\bullet\bullet}$$

		Fluorita					
		0%	1%	2%	3%	4%	
M	1	1.37	-1.21	-1.22	0.79	0.27	
e	2	-1.33	0.98	1.27	-0.79	-0.13	
z	3	0.84	-0.05	0.88	-0.84	-0.82	
c	4	-0.64	-0.02	-0.60	1.36	-0.10	
l	5	-0.11	0.28	-0.45	0.24	0.04	
a	6	-0.13	0.02	0.12	-0.76	0.74	

$$\hat{s}_R^2 = \frac{\sum\sum e_{ij}^2}{(I-1)(J-1)} = \frac{17.51}{20} = 0.88$$

Contraste de Hipótesis

- Si la Fluorita no influye, los I tratamientos son iguales a efectos de coste, entonces

$$\alpha_1 = \alpha_2 = \cdots = \alpha_I \qquad \sum_{i=1}^{I} \alpha_i = 0$$

$$H_0: \quad \alpha_1 = \alpha_2 = \cdots = \alpha_I = 0$$

$$H_1: \quad \text{Algún } \alpha_{\mathrm{i}} \text{ es distinto de } 0$$

Análisis de la varianza

$$y_{ij} = \mu + \alpha_i + \beta_j + u_{ij} \qquad \Rightarrow \qquad y_{ij} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j + e_{ij}$$

$$y_{ij} = \bar{y}_{\bullet\bullet} + (\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet}) + (\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet}) + (y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} + \bar{y}_{\bullet\bullet})$$

$$y_{ij} - \bar{y}_{\bullet\bullet} = (\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet}) + (\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet}) + (y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} + \bar{y}_{\bullet\bullet})$$

$$\sum_{i=1}^{I}\sum_{j=1}^{I}(y_{ij} - \bar{y}_{\bullet\bullet})^2 = \sum_{i=1}^{I}\sum_{j=1}^{J}(\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2 + \sum_{i=1}^{I}\sum_{j=1}^{J}(\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet})^2 + \sum_{i=1}^{I}\sum_{j=1}^{J} e_{ij}^2$$

$$\boxed{\sum_{i=1}^{I}\sum_{j=1}^{I}(y_{ij} - \bar{y}_{\bullet\bullet})^2 = J\sum_{i=1}^{I}(\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2 + I\sum_{j=1}^{J}(\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet})^2 + \sum_{i=1}^{I}\sum_{j=1}^{J} e_{ij}^2}$$

Variabilidades

$$VT = \sum_{i=1}^{I} \sum_{j=1}^{I} (y_{ij} - \bar{y}_{\bullet\bullet})^2$$

$$VE(T) = J \sum_{i=1}^{I} (\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2$$

$$VE(B) = I \sum_{j=1}^{J} (\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet})^2$$

$$VNE = \sum_{i=1}^{I} \sum_{j=1}^{J} e_{ij}^2$$

$$VT = VE(\mathrm{T}) + VE(\mathrm{B}) + VNE$$

$$(n-1) = (I-1) + (J-1) + (I-1)(J-1)$$

Contraste sobre tratamientos

$$H_0: \quad \alpha_1 = \alpha_2 = \cdots = \alpha_I = 0$$
$$H_1: \quad \text{Algún } \alpha_{\text{i}} \text{ es distinto de } 0$$

$$\hat{s}_R^2 = \frac{VNE}{(I-1)(J-1)} \qquad \rightarrow \qquad E[\hat{s}_R^2] = \sigma^2$$

$$\text{Si Ho es cierto, } \hat{s}_T^2 = \frac{VE(\text{Tratamientos})}{I-1} \rightarrow E[\hat{s}_T^2] = \sigma^2$$

$$F_T = \frac{\hat{s}_T^2}{\hat{s}_R^2} = \frac{J\sum_{i=1}^{I}(\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2 \big/ I-1}{\hat{s}_R^2} \rightarrow F_{I-1;(I-1)(J-1)}$$

$$\text{Si } F_T > F_\alpha \Rightarrow \text{ Se rechaza Ho}$$

Explicación del contraste

Si Ho es cierto $\alpha_i = 0 \quad \Rightarrow \quad y_{ij} \to N(\mu + \beta_j, \sigma^2)$

$$\overline{y}_{i\bullet} = \frac{y_{i1} + y_{i2} + \cdots + y_{iJ}}{J}, \quad E[\overline{y}_{i\bullet}] = \frac{J\mu + \sum_{j=1}^{J} \beta_j}{J} = \mu$$

$$\overline{y}_{1\bullet}, \overline{y}_{2\bullet}, \ldots, \overline{y}_{I\bullet} \to N(\mu, \frac{\sigma^2}{J})$$

$$\boxed{\overline{y}_{\bullet\bullet} = \frac{\overline{y}_{1\bullet} + \overline{y}_{2\bullet} + \cdots + \overline{y}_{I\bullet}}{I} \Rightarrow \hat{s}_T^2 = \frac{J\sum_{i=1}^{I}(\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet})^2}{I-1} \Rightarrow E\left[\frac{J\sum_{i=1}^{I}(\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet})^2}{I-1}\right] = \sigma^2}$$

- Cuando Ho es cierto, $\hat{s}_T^2$ y $\hat{s}_R^2$ serán parecidas.
- Cuando Ho es falso, $\hat{s}_T^2$ será mayor que $\hat{s}_R^2$.

Contraste de bloques

$$\begin{array}{ll} H_0: & \beta_1 = \beta_2 = \cdots = \beta_J = 0 \\ H_1: & \text{Algún } \beta_{\text{j}} \text{ es distinto de } 0 \end{array}$$

$$\text{Si Ho es cierto, } \hat{s}_B^2 = \frac{VE(\text{Bloques})}{J-1} \rightarrow E[\hat{s}_B^2] = \sigma^2$$

$$F_B = \frac{\hat{s}_B^2}{\hat{s}_R^2} = \frac{I\sum_{j=1}^{J}(\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet})^2 \Big/ J-1}{\hat{s}_R^2} \rightarrow F_{J-1;(I-1)(J-1)}$$

$$\text{Si } F_B > F_\alpha \Rightarrow \text{ Se rechaza Ho}$$

Tabla de análisis de la varianza

Fuentes Variabilidad	Suma de Cuadrados	Grados de Libertad.	Varianza	F	$\mathrm{p-valor}$
Tratamiento	$J\Sigma(\bar{y}_{i\bullet}-\bar{y}_{\bullet\bullet})^2$	$I-1$	$\hat{s}_T^2$	$\hat{s}_T^2 / \hat{s}_R^2$	p_T
Bloque	$I\Sigma(\bar{y}_{\bullet j}-\bar{y}_{\bullet\bullet})^2$	$J-1$	$\hat{s}_B^2$	$\hat{s}_B^2 / \hat{s}_R^2$	p_B
Residual	$\Sigma\Sigma e_{ij}^2$	$(I-1)(J-1)$	$\hat{s}_R^2$		
Total	$\Sigma\Sigma(y_{ij}-\bar{y}_{\bullet\bullet})^2$	n - 1			

Tabla de análisis de la varianza

Fuentes Variabilidad	Suma de Cuadrados.	Grados Libertad.	Varianza	F	p – valor
Tratamiento	26.05	4	6.51	7.4	.0008
Bloque	247.77	5	49.55	56.6	.0000
Residual	17.51	20	0.88		
Total	291.33	29			

Sin bloques

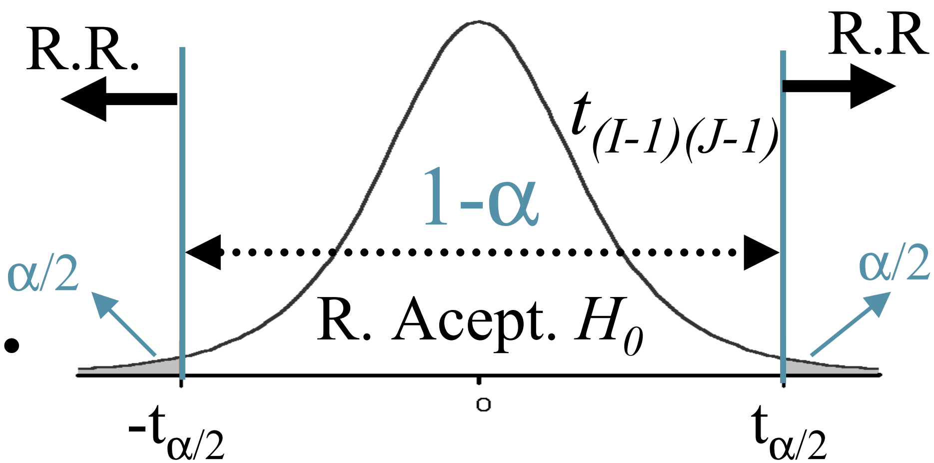
Fuentes Variabilidad	Suma de Cuadrados.	Grados Libertad.	Varianza	F	p – valor
Tratamiento	26.05	4	6.51	0.61	.8887
Residual	265.28	25	10.61		
Total	291.33	29			

Contrastes múltiples: tratamientos

$$H_0: \quad \alpha_i = \alpha_j$$

$$H_1: \quad \alpha_i \neq \alpha_j$$

$$\left.\begin{array}{l} \hat{\alpha}_i = \bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet} \\ \hat{\alpha}_j = \bar{y}_{j\bullet} - \bar{y}_{\bullet\bullet} \end{array}\right\} \quad \hat{\alpha}_i - \hat{\alpha}_j = \bar{y}_{i\bullet} - \bar{y}_{j\bullet}$$



$$\hat{\alpha}_i - \hat{\alpha}_j \to N(\alpha_i - \alpha_j, \frac{\sigma^2}{J} + \frac{\sigma^2}{J})$$

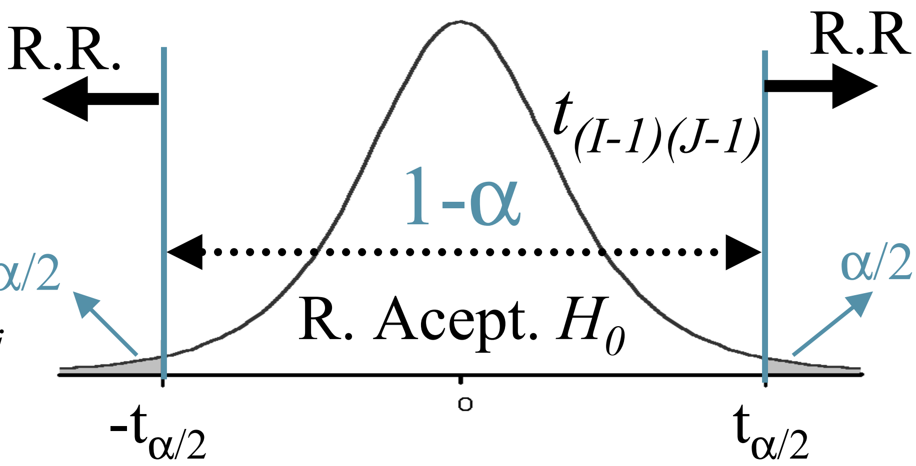
$$\frac{\bar{y}_{i\bullet} - \bar{y}_{j\bullet}}{\hat{s}_R \sqrt{\dfrac{2}{J}}} \to t_{(I-1)(J-1)}$$

$$\left|\bar{y}_{i\bullet} - \bar{y}_{j\bullet}\right| > \underbrace{t_{\alpha/2}\hat{s}_R\sqrt{\frac{2}{J}}}_{LSD} \Rightarrow \text{Se rechaza } H_0$$

Contrastes múltiples: bloques

$$H_0: \quad \beta_i = \beta_j$$

$$H_1: \quad \beta_i \neq \beta_j$$



$$\left.\begin{aligned} \hat{\beta}_i &= \bar{y}_{\bullet i} - \bar{y}_{\bullet\bullet} \\ \hat{\beta}_j &= \bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet} \end{aligned}\right\} \quad \hat{\beta}_i - \hat{\beta}_j = \bar{y}_{\bullet i} - \bar{y}_{\bullet j}$$

$$\hat{\beta}_i - \hat{\beta}_j \to N\left(\beta_i - \beta_j, \frac{\sigma^2}{I} + \frac{\sigma^2}{I}\right)$$

$$\frac{\bar{y}_{\bullet i} - \bar{y}_{\bullet j}}{\hat{s}_R \sqrt{\frac{2}{I}}} \to t_{(I-1)(J-1)}$$

$$\left|\bar{y}_{\bullet i} - \bar{y}_{\bullet j}\right| > \underbrace{t_{\alpha/2}\hat{s}_R\sqrt{\frac{2}{I}}}_{LSD} \Rightarrow \text{Se rechaza } H_0$$

Comparación de medias

Fluorita

$$LSD = t_{\alpha/2}\hat{s}_R\sqrt{\frac{2}{J}}$$

$$= 2.085 \times 0.93 \times \sqrt{\frac{2}{6}}$$

$$= 1.13$$

	LSD = 1.13				
	0%	**1%**	**2%**	**3%**	**4%**
0%	0	0.58	**2.49**	**1.99**	0.69
1%		0	**1.90**	**1.40**	**-1.30**
2%			0	-0.50	**-1.80**
3%				0	**-1.30**
4%					0

Mezcla

$$LSD = t_{\alpha/2}\hat{s}_R\sqrt{\frac{2}{I}}$$

$$= 2.085 \times 0.93 \times \sqrt{\frac{2}{5}}$$

$$= 1.24$$

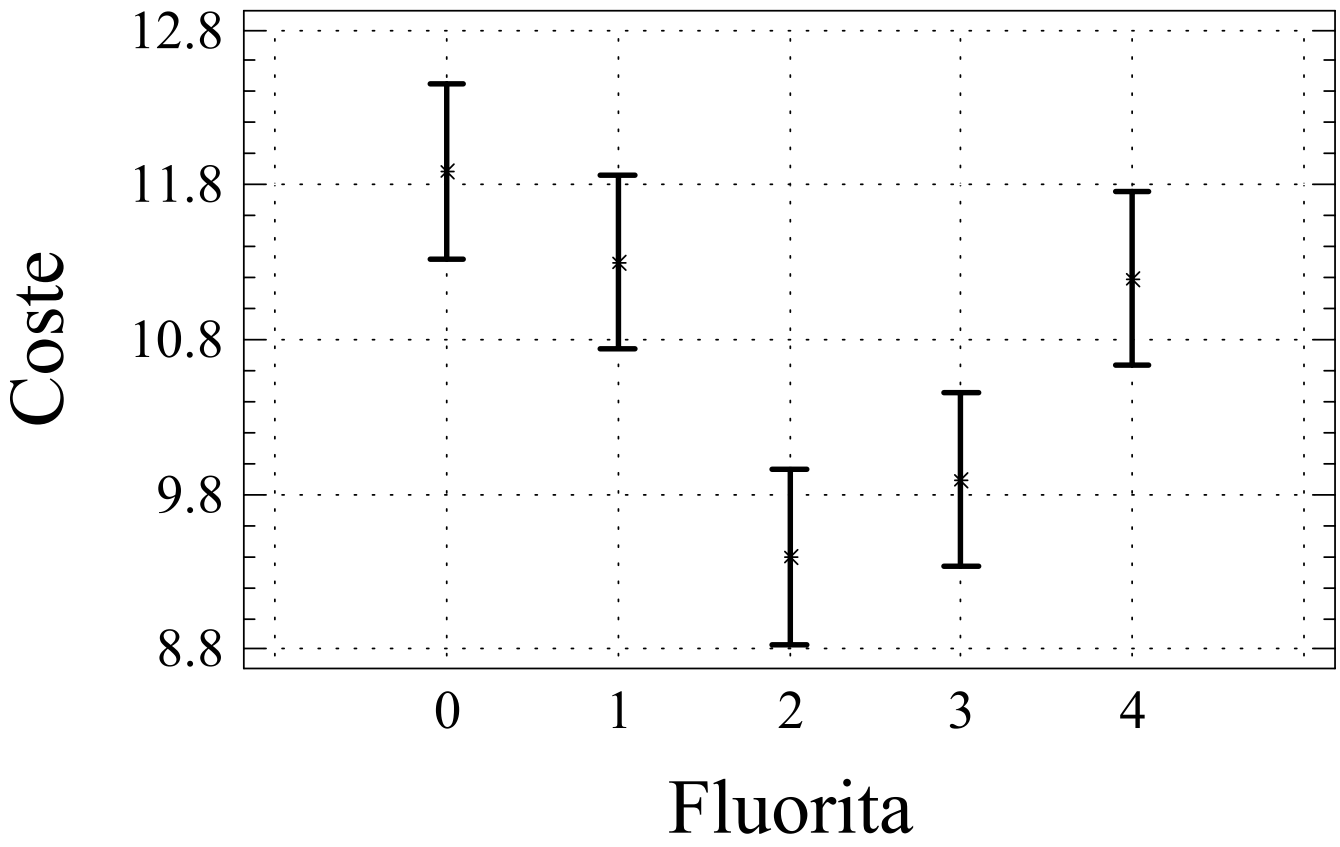
	LSD=1.24					
	1	**2**	**3**	**4**	**5**	**6**
1	0.00	**3.90**	**-3.82**	**2.52**	**3.76**	**4.24**
2		0	**6.60**	**5.60**	**4.60**	**3.60**
3			0	**6.34**	**7.58**	**8.07**
4				0	1.23	**1.72**
5					0	0.49
6						0

Intervalos de confianza (ejemplo)

$$\mu + \alpha_i \in \overline{y}_{i\bullet} \pm t_{\alpha/2} \frac{\hat{s}_R}{\sqrt{J}}$$

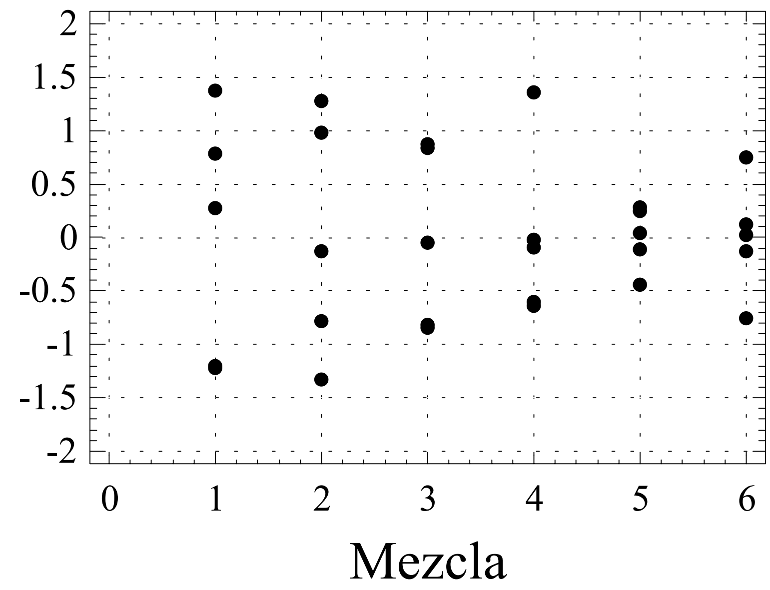
Fluorita	Medias	L.inf.	L.Sup.
0%	11.88	11.09	12.68
1%	11.30	10.50	12.10
2%	9.40	8.60	10.19
3%	9.90	9.10	10.69
4%	11.19	10.40	11.99

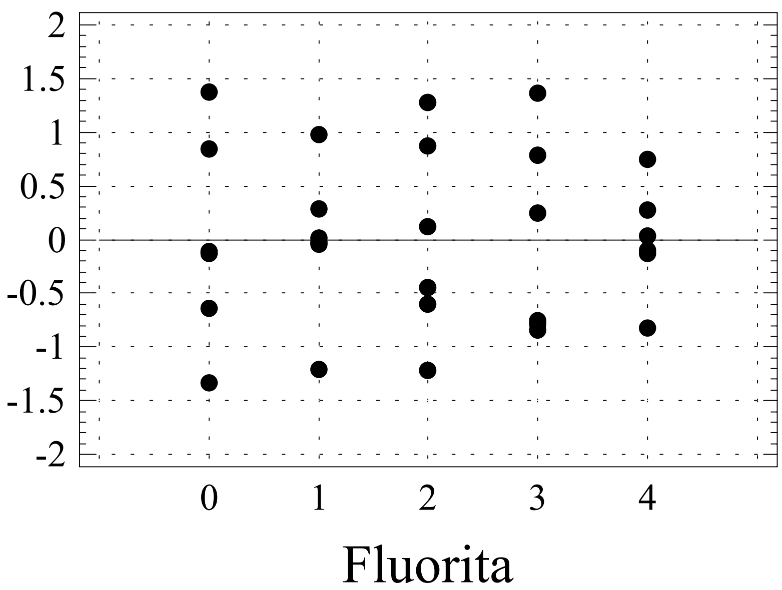
Intervalos para las medias 95%

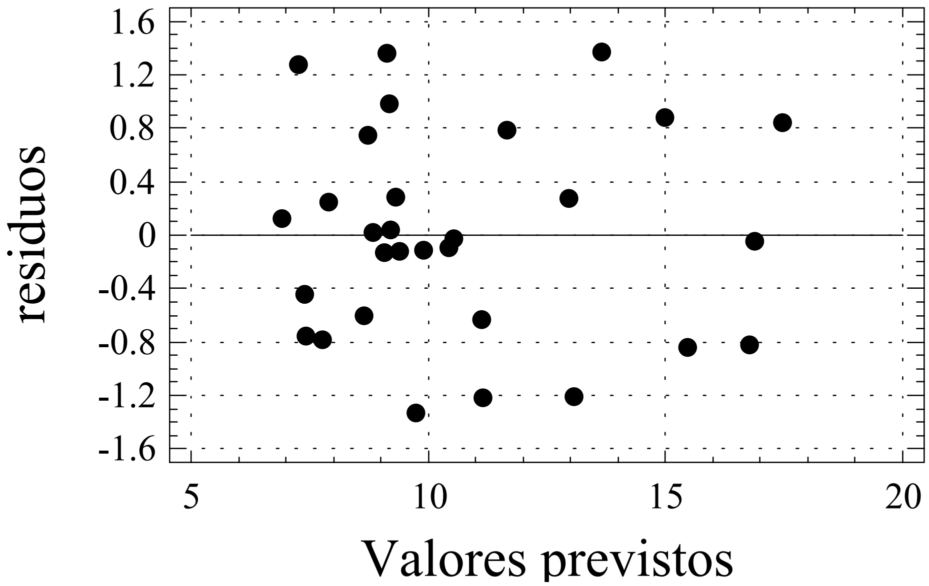


Diagnosis: Homocedasticidad

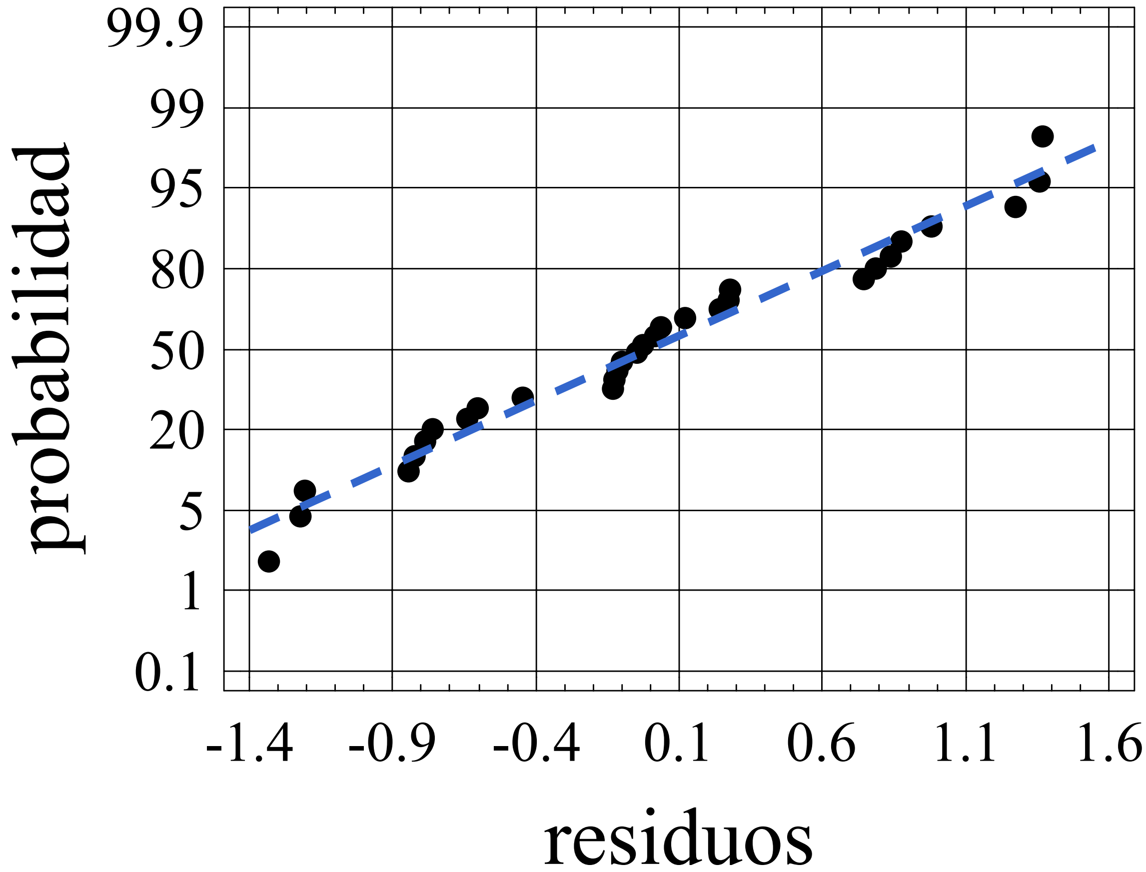
Gráfico de residuos







Diagnosis: normalidad



Diseños factoriales

		ANTÍDOTO			
		A	B	C	D
VENENOS	I	0.31	0.82	0.43	0.45
		0.45	1.10	0.45	0.71
		0.46	0.88	0.63	0.66
		0.43	0.72	0.72	0.62
	II	0.36	0.92	0.44	0.56
		0.29	0.61	0.35	1.02
		0.40	0.49	0.31	0.71
		0.23	1.24	0.40	0.38
	III	0.22	0.30	0.23	0.30
		0.21	0.37	0.25	0.36
		0.18	0.38	0.24	0.31
		0.23	0.29	0.22	0.33

Se analiza el efecto de tres venenos y cuatro antídotos en el tiempo de supervivencia de unas ratas.

Modelo

Factor A

Factor B

	1	2	$\cdots$	I
1	y_{111} y_{112} $\vdots$ y_{11m}	y_{211} y_{212} $\vdots$ y_{21m}	$\cdots$	y_{I11} y_{I12} $\vdots$ y_{I1m}
2	y_{121} y_{122} $\vdots$ y_{12m}	y_{221} y_{222} $\vdots$ y_{22m}	$\cdots$	y_{I21} y_{I22} $\vdots$ y_{I2m}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
J	y_{1J1} y_{1J2} $\vdots$ y_{1Jm}	y_{2J1} y_{2J2} $\vdots$ y_{2Jm}	$\cdots$	y_{IJ1} y_{IJ2} $\vdots$ y_{IJm}

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + u_{ijk}$$

- Normalidad
- Independencia
- Homocedasticidad

I×J tratamientos

m replicaciones

$$n = m \times I \times J$$

Modelo

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + u_{ijk}$$

$\sum_{i=1}^{I} \alpha_i = 0$ $\sum_{j=1}^{J} \beta_j = 0$ $\sum_{i=1}^{I} \alpha\beta_{ij} = 0, \quad \forall j$

$\sum_{j=1}^{J} \alpha\beta_{ij} = 0, \quad \forall i$

μ : Media global

α_i : Efecto del factor A i, i=1,...,I

β_j : Efecto del factor B j, j=1,2,...,J

$\alpha\beta_{ij}$: Interacción de niveles ij

u_{ij} : Componente aleatoria $N(0,\sigma^2)$

Estimación del modelo

$\mu :\rightarrow 1$

$\alpha_{\text{i}} :\rightarrow I-1$

$\beta_{\text{j}} :\rightarrow J-1$

$\alpha\beta_{ij} :\rightarrow (I-1)(J-1)$

$\sigma^2 :\rightarrow 1$

$$\hat{\mu} = \overline{y}_{\bullet\bullet\bullet}$$

$$\hat{\alpha}_i = \overline{y}_{i\bullet\bullet} - \overline{y}_{\bullet\bullet\bullet}$$

$$\hat{\beta}_j = \overline{y}_{\bullet j \bullet} - \overline{y}_{\bullet\bullet\bullet}$$

$$\widehat{\alpha\beta}_{ij} = \overline{y}_{ij\bullet} - \overline{y}_{i\bullet\bullet} - \overline{y}_{\bullet j\bullet} + \overline{y}_{\bullet\bullet\bullet}$$

$$\hat{\sigma}^2 = \hat{s}_R^2 = \frac{\sum\sum e_{ij}^2}{IJ(m-1)}$$

$$\overline{y}_{ij\bullet} = \frac{\sum_{k=1}^{m} y_{ijk}}{m} \qquad \overline{y}_{i\bullet\bullet} = \frac{\sum_{j=1}^{J}\sum_{k=1}^{m} y_{ijk}}{mJ} \qquad \overline{y}_{\bullet j\bullet} = \frac{\sum_{i=1}^{I}\sum_{k=1}^{m} y_{ijk}}{mI} \qquad \overline{y}_{\bullet\bullet\bullet} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{m} y_{ijk}}{n}$$

Estimación

		ANTÍDOTO			
		A	B	C	D
VENENOS	I	0.31	0.82	0.43	0.45
		0.45	1.10	0.45	0.71
		0.46	0.88	0.63	0.66
		0.43	0.72	0.72	0.62
		0.41	**0.88**	**0.56**	**0.61**
	II	0.36	0.92	0.44	0.56
		0.29	0.61	0.35	1.02
		0.40	0.49	0.31	0.71
		0.23	1.24	0.40	0.38
		0.32	**0.82**	**0.38**	**0.67**
	III	0.22	0.30	0.23	0.30
		0.21	0.37	0.25	0.36
		0.18	0.38	0.24	0.31
		0.23	0.29	0.22	0.33
		0.21	**0.34**	**0.24**	**0.33**

Estimación

		ANTÍDOTO					$\hat{\alpha}_i$
		A	**B**	**C**	**D**	**Medias**	
VENENOS	**I**	0.31 0.45 0.46 0.43	0.82 1.10 0.88 0.72	0.43 0.45 0.63 0.72	0.45 0.71 0.66 0.62	0.615	0.136
	Medias	0.41	0.88	0.56	0.61		
	$\alpha\beta_{ij}$	**-0.038**	**0.067**	**0.032**	**-0.061**		
	II	0.36 0.29 0.40 0.23	0.92 0.61 0.49 1.24	0.44 0.35 0.31 0.40	0.56 1.02 0.71 0.38	0.544	0.066
	Medias	0.32	0.82	0.38	0.67		
	$\alpha\beta_{ij}$	**-0.060**	**0.073**	**-0.080**	**0.068**		
	III	0.22 0.21 0.18 0.23	0.30 0.37 0.38 0.29	0.23 0.25 0.24 0.22	0.30 0.36 0.31 0.33	0.276	-0.202
	Medias	0.21	0.34	0.24	0.33		
	$\alpha\beta_{ij}$	**0.098**	**-0.139**	**0.048**	**-0.007**		
	Medias	0.314	0.677	0.389	0.534	0.479	
	$\hat{\beta}_j$	**-0.164**	**0.198**	**-0.089**	**0.056**		

Residuos

RESIDUOS					
		ANTÍDOTO			
		A	**B**	**C**	**D**
VENENOS	**I**	-0.103	-0.060	-0.128	-0.160
		0.038	0.220	-0.108	0.100
		0.048	0.000	0.073	0.050
		0.018	-0.160	0.163	0.010
		0.00	**0.00**	**0.00**	**0.00**
	II	0.040	0.105	0.065	-0.108
		-0.030	-0.205	-0.025	0.353
		0.080	-0.325	-0.065	0.043
		-0.090	0.425	0.025	-0.288
		0.00	**0.00**	**0.00**	**0.00**
	III	0.010	-0.035	-0.005	-0.025
		0.000	0.035	0.015	0.035
		-0.030	0.045	0.005	-0.015
		0.020	-0.045	-0.015	0.005
		0.00	**0.00**	**0.00**	**0.00**

Análisis de la varianza

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + u_{ijk} \qquad \Rightarrow \qquad y_{ijk} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j + \widehat{\alpha\beta}_{ij} + e_{ijk}$$

$$y_{ijk} = \bar{y}_{\bullet\bullet\bullet} + (\bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet}) + (\bar{y}_{\bullet j\bullet} - \bar{y}_{\bullet\bullet\bullet}) + (\bar{y}_{ij\bullet} - \bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet j\bullet} + \bar{y}_{\bullet\bullet\bullet}) + (y_{ijk} - \bar{y}_{ij\bullet})$$

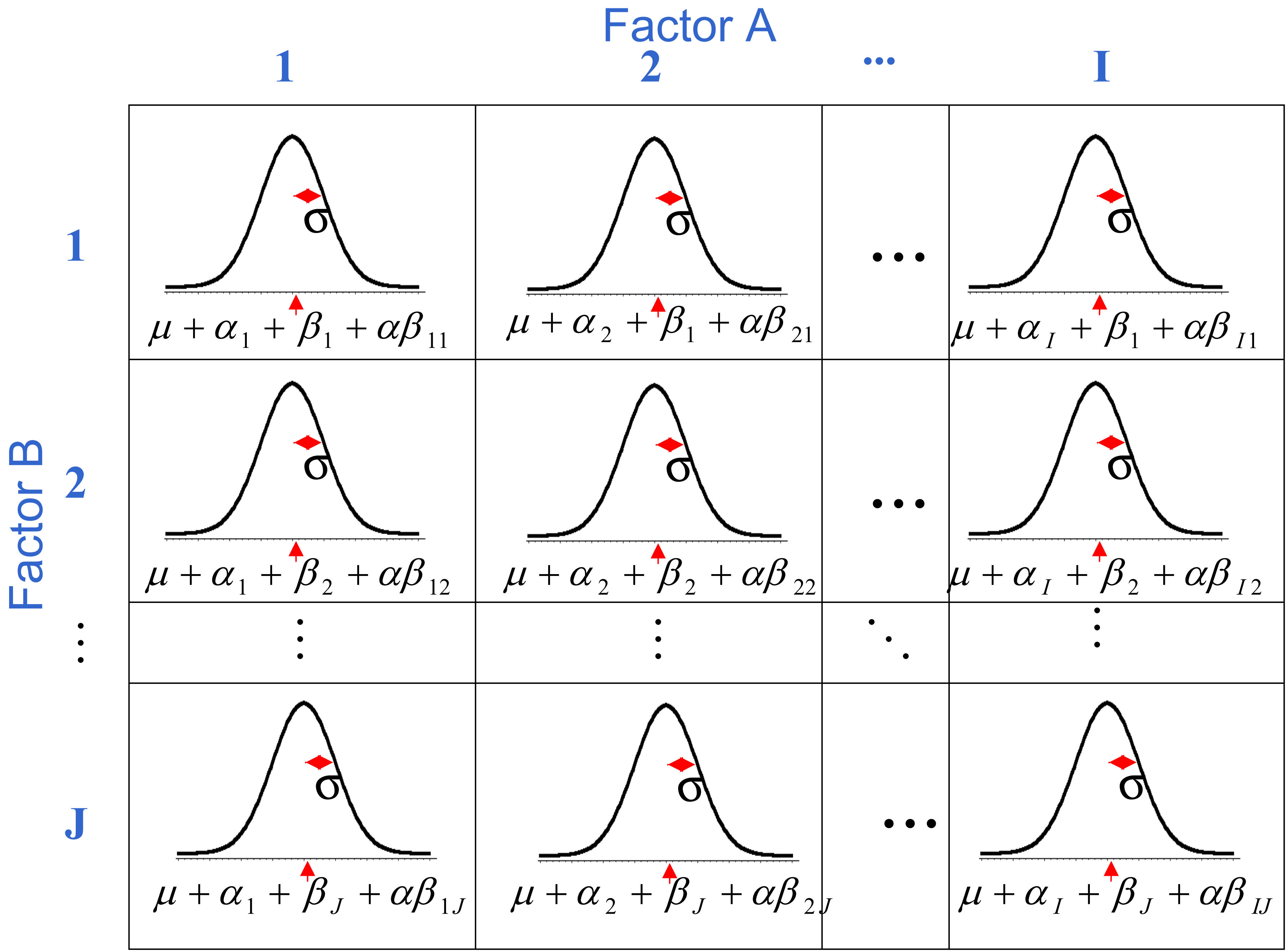
$$y_{ijk} - \bar{y}_{\bullet\bullet\bullet} = (\bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet}) + (\bar{y}_{\bullet j\bullet} - \bar{y}_{\bullet\bullet\bullet}) + (\bar{y}_{ij\bullet} - \bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet j\bullet} + \bar{y}_{\bullet\bullet\bullet}) + e_{ijk}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{m}(y_{ijk} - \bar{y}_{\bullet\bullet\bullet})^2 = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{m}(\bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet\bullet})^2 + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{m}(\bar{y}_{\bullet j\bullet} - \bar{y}_{\bullet\bullet})^2 +$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{m}(\bar{y}_{ij\bullet} - \bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet j\bullet} + \bar{y}_{\bullet\bullet\bullet})^2 + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{m} e_{ij}^2$$

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{m}(y_{ijk} - \bar{y}_{\bullet\bullet\bullet})^2 = mJ\sum_{i=1}^{I}(\bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet\bullet})^2 + mI\sum_{j=1}^{J}(\bar{y}_{\bullet j\bullet} - \bar{y}_{\bullet\bullet})^2$$

$$+ m\sum_{i=1}^{I}\sum_{j=1}^{J}(\bar{y}_{ij\bullet} - \bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet j\bullet} + \bar{y}_{\bullet\bullet\bullet})^2 + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{m} e_{ij}^2$$


Factor A
1
2
...
I
Factor B
1
2
⋮
J
σ
$\mu + \alpha_1 + \beta_1 + \alpha\beta_{11}$
$\mu + \alpha_2 + \beta_1 + \alpha\beta_{21}$
$\mu + \alpha_I + \beta_1 + \alpha\beta_{I1}$
$\mu + \alpha_1 + \beta_2 + \alpha\beta_{12}$
$\mu + \alpha_2 + \beta_2 + \alpha\beta_{22}$
$\mu + \alpha_I + \beta_2 + \alpha\beta_{I2}$
$\mu + \alpha_1 + \beta_J + \alpha\beta_{1J}$
$\mu + \alpha_2 + \beta_J + \alpha\beta_{2J}$
$\mu + \alpha_I + \beta_J + \alpha\beta_{IJ}$

Variabilidades

$$VT = \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{m}(y_{ijk} - \overline{y}_{\bullet\bullet\bullet})^2$$

$$VE(A) = mJ\sum_{i=1}^{I}(\overline{y}_{i\bullet\bullet} - \overline{y}_{\bullet\bullet\bullet})^2$$

$$VE(B) = mI\sum_{j=1}^{J}(\overline{y}_{\bullet j\bullet} - \overline{y}_{\bullet\bullet\bullet})^2$$

$$VE(A\times B) = m\sum_{i=1}^{I}\sum_{j=1}^{J}(\overline{y}_{ij\bullet} - \overline{y}_{i\bullet\bullet} - \overline{y}_{\bullet j\bullet} + \overline{y}_{\bullet\bullet\bullet})^2$$

$$VNE = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{m}(y_{ijk} - \overline{y}_{ij\bullet})^2$$

$$VT = VE(A) + VE(B) + VE(A\times B) \quad + \quad VNE$$

$$(n-1) = (I-1) + (J-1) + (I-1)(J-1) + IJ(m-1)$$

Contraste efecto principal de **factor A**

$$\boxed{\begin{array}{l} H_0: \quad \alpha_1 = \alpha_2 = \cdots = \alpha_I = 0 \\ H_1: \quad \text{Algún } \alpha_\mathrm{i} \text{ es distinto de } 0 \end{array}}$$

$$\hat{s}_R^2 = \frac{VNE}{IJ(m-1)} \qquad \rightarrow \qquad E[\hat{s}_R^2] = \sigma^2$$

Si Ho es cierto, $\hat{s}_A^2 = \dfrac{VE(A)}{I-1} \rightarrow E[\hat{s}_A^2] = \sigma^2$

$$\boxed{F_A = \frac{\hat{s}_A^2}{\hat{s}_R^2} = \frac{mJ\sum_{i=1}^{I}(\bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet})^2 \Big/ I-1}{\hat{s}_R^2} \rightarrow F_{I-1;IJ(m-1)}}$$

Si $F_A > F_\alpha \Rightarrow$ Se rechaza Ho

Contraste efecto principal de **factor B**

$$\boxed{\begin{aligned} &H_0: \quad \beta_1 = \beta_2 = \cdots = \beta_J = 0 \\ &H_1: \quad \text{Algún } \beta_\text{j} \text{ es distinto de 0} \end{aligned}}$$

Si Ho es cierto, $\hat{s}_B^2 = \dfrac{VE(B)}{J-1} \rightarrow E[\hat{s}_B^2] = \sigma^2$

$$\boxed{F_B = \frac{\hat{s}_B^2}{\hat{s}_R^2} = \frac{mI \sum_{j=1}^{J} (\bar{y}_{\bullet j \bullet} - \bar{y}_{\bullet\bullet\bullet})^2 \Big/ J-1}{\hat{s}_R^2} \rightarrow F_{J-1;IJ(m-1)}}$$

$$\text{Si } F_B > F_\alpha \Rightarrow \text{ Se rechaza Ho}$$

Contraste interacción **AxB**

$$H_0: \quad \alpha\beta_{11} = \alpha\beta_{12} = \cdots = \alpha\beta_{IJ} = 0$$

H_1: Algún $\alpha\beta_{ij}$ es distinto de 0

Si Ho es cierto, $\hat{s}^2_{AB} = \dfrac{VE(A \times B)}{(I-1)(J-1)} \rightarrow E[\hat{s}^2_{AB}] = \sigma^2$

$$F_{AB} = \frac{\hat{s}^2_{AB}}{\hat{s}^2_R} \rightarrow F_{(I-1)(J-1);IJ(m-1)}$$

Si $F_{AB} > F_\alpha \Rightarrow$ $\underbrace{\text{Se rechaza Ho}}_{\text{A y B interaccionan}}$

Tabla de análisis de la varianza

Fuentes Variabilidad	Suma de Cuadrados	Grados de Libertad.	Varianza	F	$p-\text{valor}$
A	$mJ\sum(\bar{y}_{i\bullet\bullet}-\bar{y}_{\bullet\bullet})^2$	$I-1$	$\hat{s}_T^2$	$\hat{s}_A^2 / \hat{s}_R^2$	p_T
B	$mI\sum(\bar{y}_{\bullet j\bullet}-\bar{y}_{\bullet\bullet})^2$	$J-1$	$\hat{s}_B^2$	$\hat{s}_B^2 / \hat{s}_R^2$	p_B
$\text{A}\times\text{B}$	$m\sum\sum(y_{ij\bullet}-\bar{y}_{i\bullet\bullet}-\bar{y}_{\bullet j\bullet}+\bar{y}_{\bullet\bullet\bullet})^2$	$(I-1)(J-1)$	$\hat{s}_{AB}^2$	$\hat{s}_{AB}^2 / \hat{s}_R^2$	p_{AB}
Residual	$\sum\sum\sum e_{ijk}^2$	$IJ(m-1)$	$\hat{s}_R^2$		
Total	$\sum\sum\sum(y_{ijk}-\bar{y}_{\bullet\bullet\bullet})^2$	$n-1$			

Tabla de análisis de la varianza

Fuentes Variabilidad	Suma de Cuadrados.	Grados Libertad.	Varianza	F	$p-\text{valor}$
Veneno	1.033	2	0.516	23.2	.0000
Antídoto	0.921	3	0.307	13.8	.0000
$\text{Ven}\times\text{Ant}$	0.250	6	0.041	1.87	.1123
Residual	0.801	36	0.022		
Total	3.005	47			

Contraste múltiples: Factor A

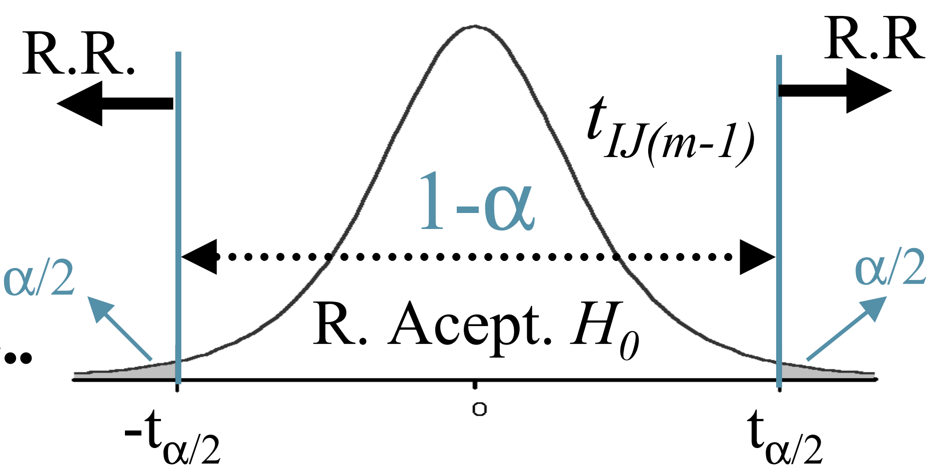
$$H_0: \quad \alpha_i = \alpha_j$$

$$H_1: \quad \alpha_i \neq \alpha_j$$

$$\left.\begin{aligned} \hat{\alpha}_i &= \bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet} \\ \hat{\alpha}_j &= \bar{y}_{j\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet} \end{aligned}\right\} \quad \hat{\alpha}_i - \hat{\alpha}_j = \bar{y}_{i\bullet\bullet} - \bar{y}_{j\bullet\bullet}$$

$$\hat{\alpha}_i - \hat{\alpha}_j \to N\left(\alpha_i - \alpha_j, \frac{\sigma^2}{mJ} + \frac{\sigma^2}{mJ}\right)$$

$$\frac{\bar{y}_{i\bullet\bullet} - \bar{y}_{j\bullet\bullet}}{\hat{s}_R \sqrt{\frac{2}{mJ}}} \to t_{IJ(m-1)}$$



$$\left|\bar{y}_{i\bullet\bullet} - \bar{y}_{j\bullet\bullet}\right| > t_{\alpha/2}\hat{s}_R \sqrt{\frac{2}{mJ}}$$

Se rechaza *Ho*

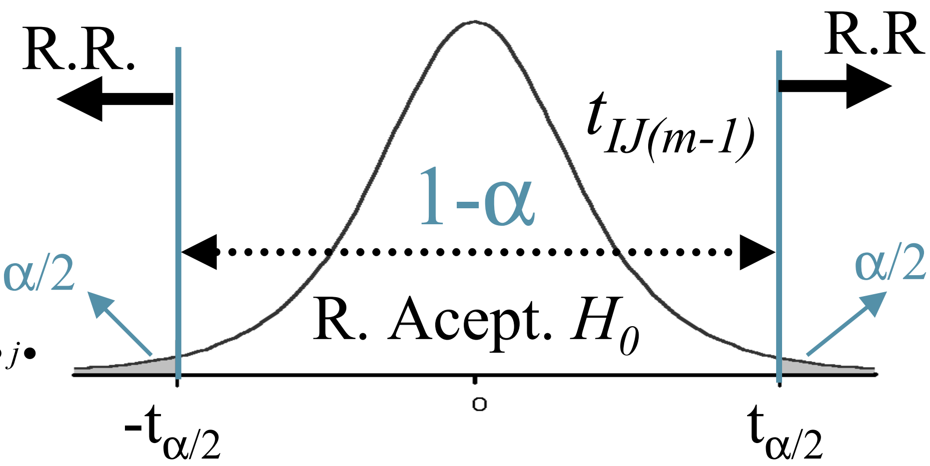
Contraste múltiples: Factor B

$$H_0: \quad \beta_i = \beta_j$$

$$H_1: \quad \beta_i \neq \beta_j$$

$$\left.\begin{array}{l}\hat{\beta}_i = \bar{y}_{\bullet i \bullet} - \bar{y}_{\bullet\bullet\bullet} \\ \hat{\beta}_j = \bar{y}_{\bullet j \bullet} - \bar{y}_{\bullet\bullet\bullet}\end{array}\right\} \quad \hat{\beta}_i - \hat{\beta}_j = \bar{y}_{\bullet i \bullet} - \bar{y}_{\bullet j \bullet}$$

$$\hat{\beta}_i - \hat{\beta}_j \to N(\beta_i - \beta_j, \frac{\sigma^2}{mI} + \frac{\sigma^2}{mI})$$



$$\frac{\bar{y}_{\bullet i \bullet} - \bar{y}_{\bullet j \bullet}}{\hat{s}_R \sqrt{\frac{2}{mI}}} \to t_{IJ(m-1)}$$

$$\left| \bar{y}_{\bullet i \bullet} - \bar{y}_{\bullet j \bullet} \right| > t_{\alpha/2} \hat{s}_R \sqrt{\frac{2}{mI}}$$

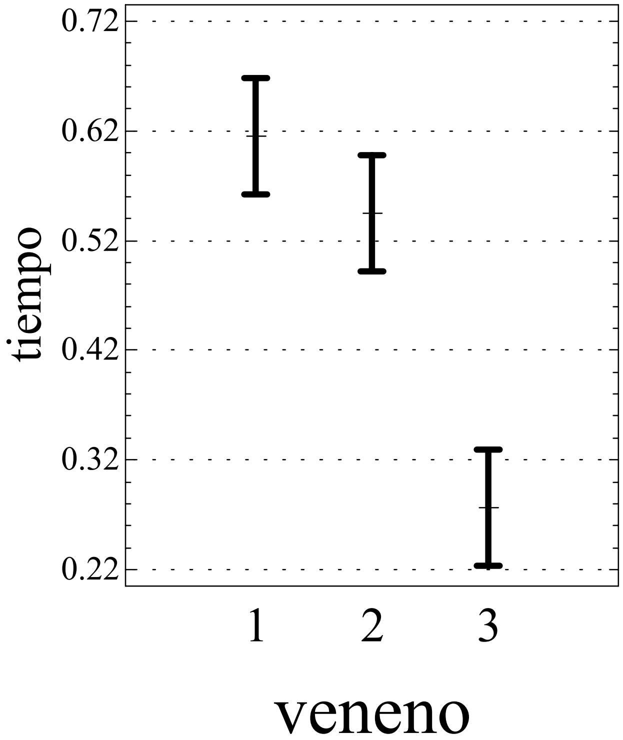
Se rechaza *Ho*

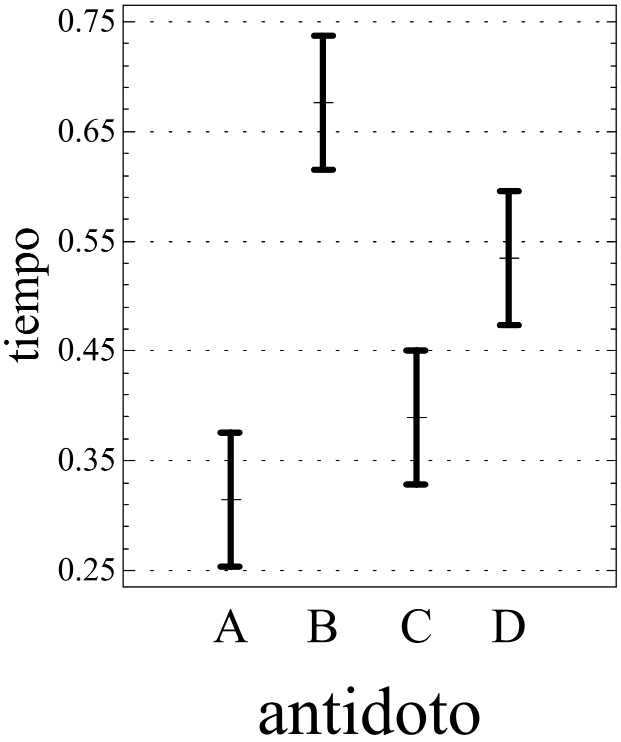
Intervalos de confianza (interacción nula)

$$\mu + \alpha_i \in \bar{y}_{i\bullet\bullet} \pm t_{\alpha/2} \frac{\hat{s}_R}{\sqrt{mJ}}$$

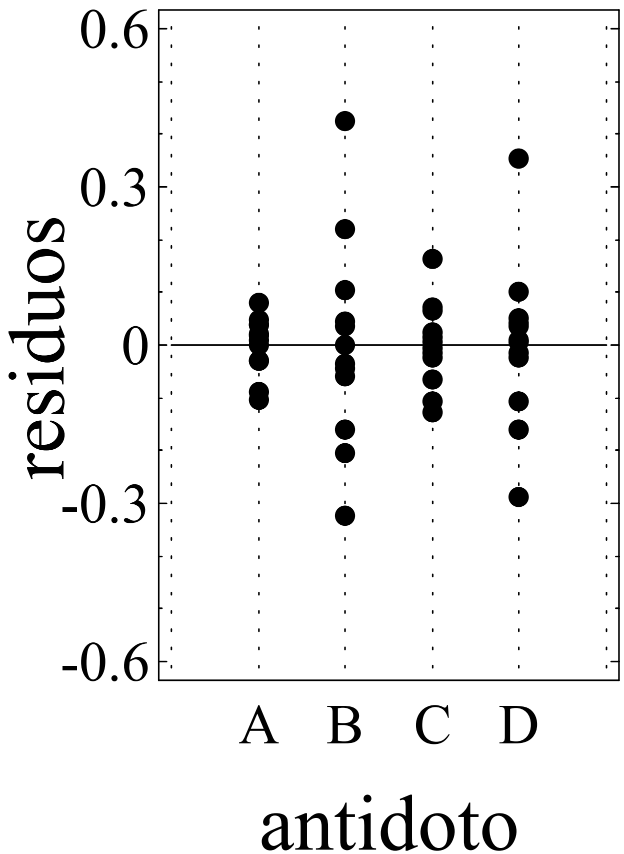
$$\mu + \beta_i \in \bar{y}_{\bullet j\bullet} \pm t_{\alpha/2} \frac{\hat{s}_R}{\sqrt{mI}}$$

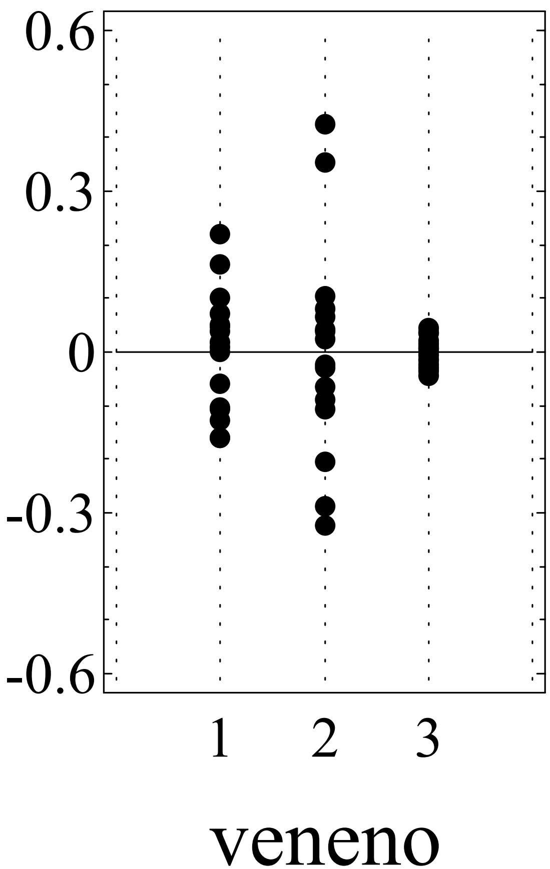
Intervalos de confianza



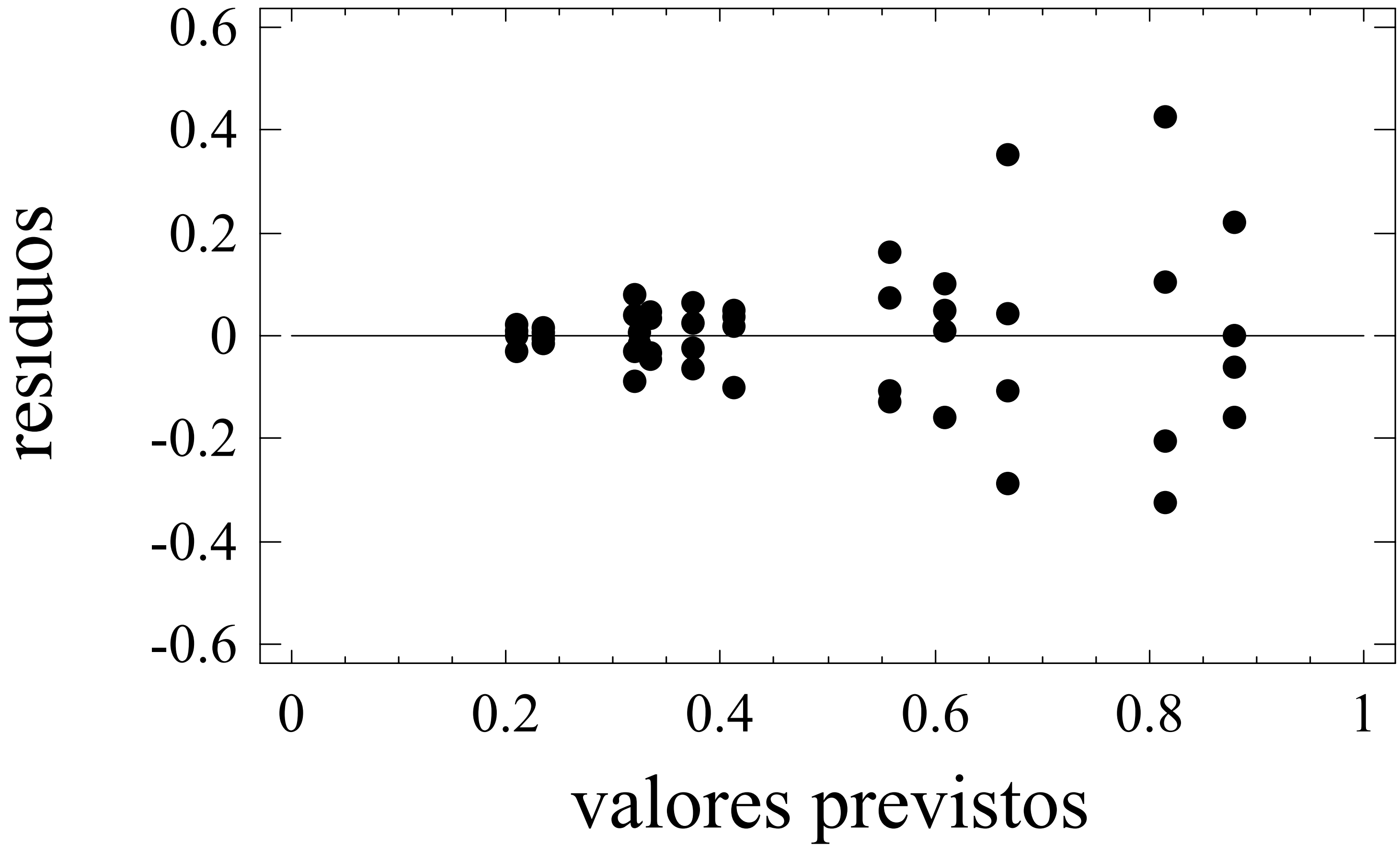


Diagnosis: homocedasticidad

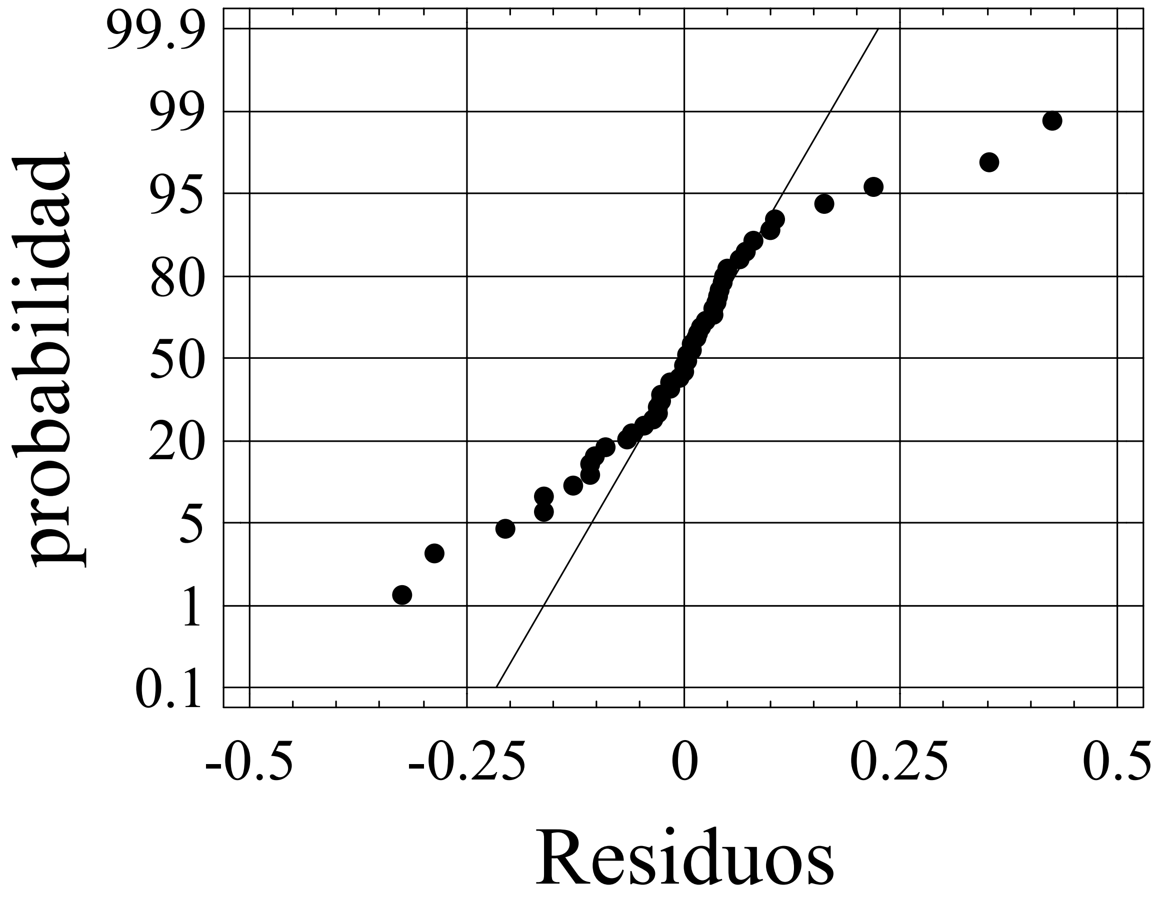




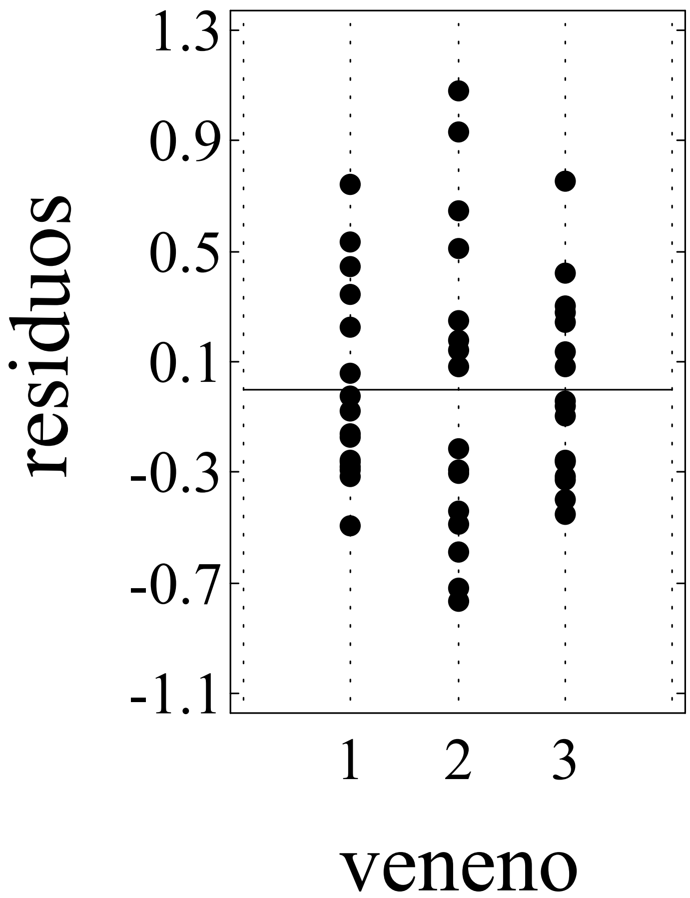
Heterocedasticidad

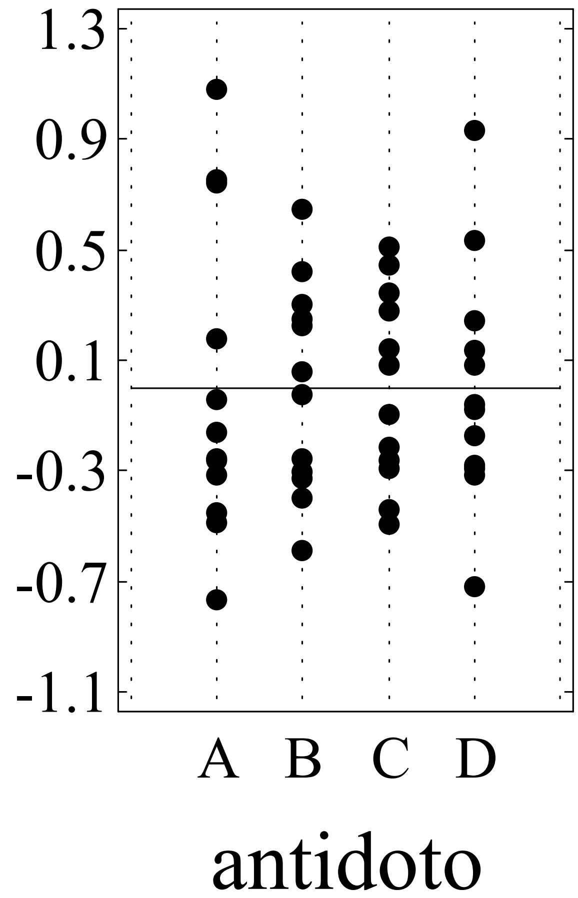


Normalidad

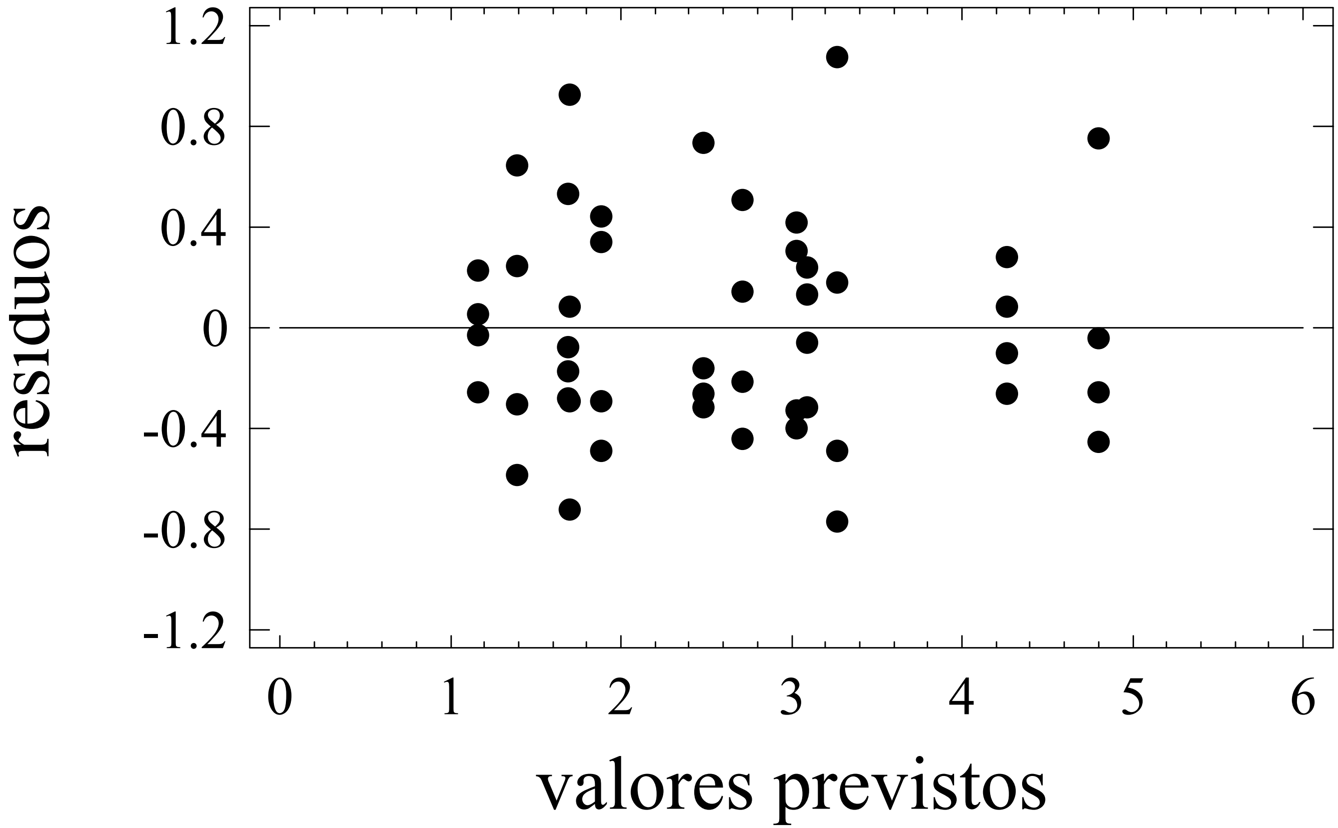


Diagnosis: homocedasticidad
datos transformados z=1/y





Datos transformados



Normalidad (datos transformados)

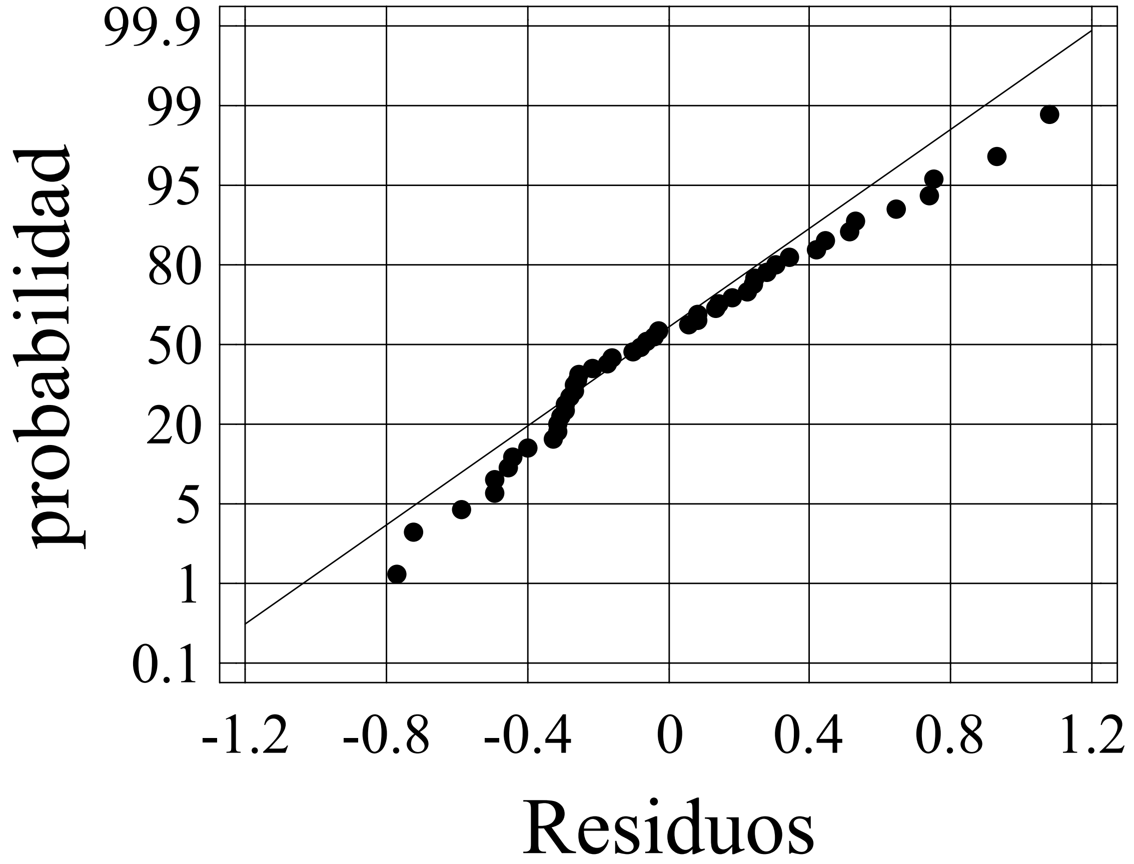


Tabla de análisis de la varianza datos transformados 1/y

Fuentes Variabilidad	Suma de Cuadrados.	Grados Libertad.	Varianza	F	p−valor
Veneno	34.87	2	17.4	72.6	.0000
Antídoto	20.41	3	6.80	28.3	.0000
Ven × Ant	1.57	6	0.26	1.09	.3867
Residual	8.68	36	0.24		
Total	65.50	47			

Comparaciones múltiples
intervalos de confianza