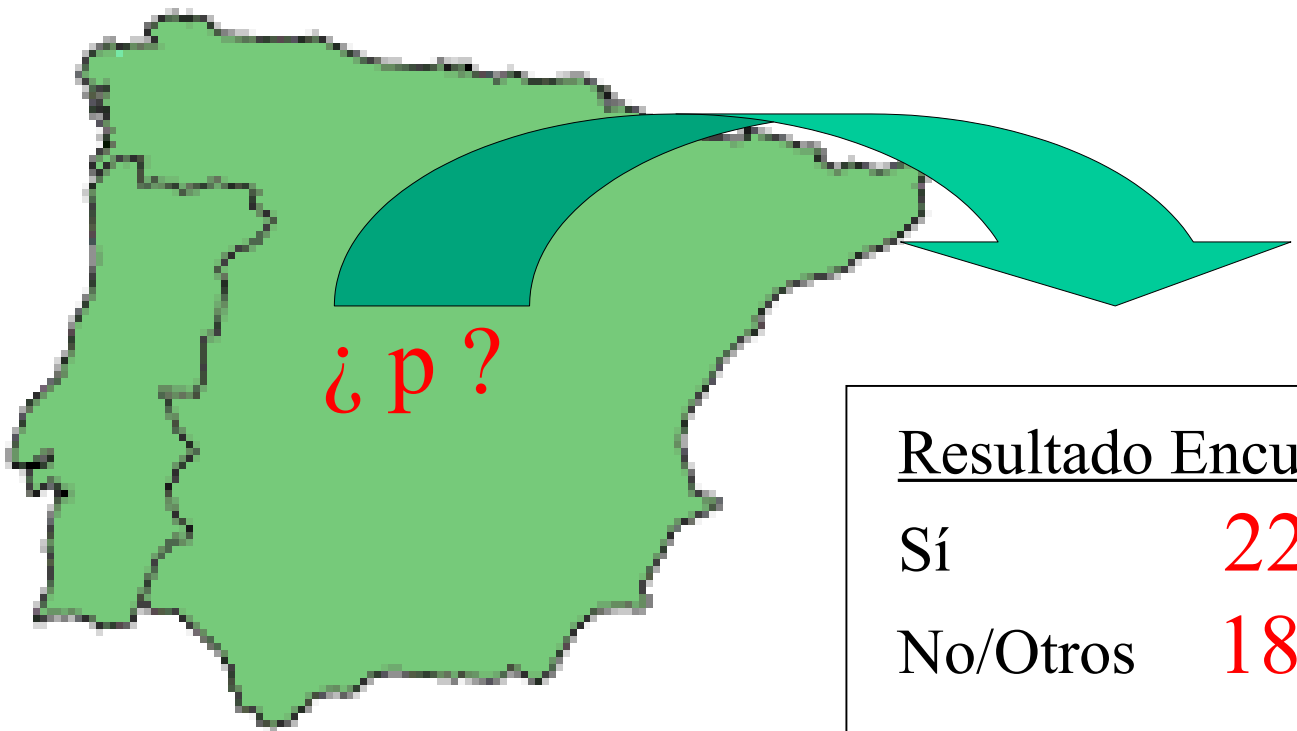
Intervalos de confianza

Concepto de intervalo de confianza

Se ha realizado una encuesta a 400 personas elegidas al azar para estimar la proporción p de votantes de un partido político.



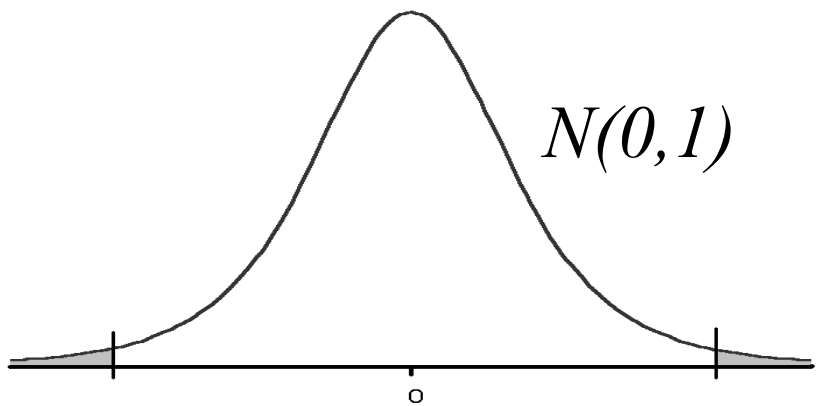
Resultado Encuesta	
Sí	220
No/Otros	180

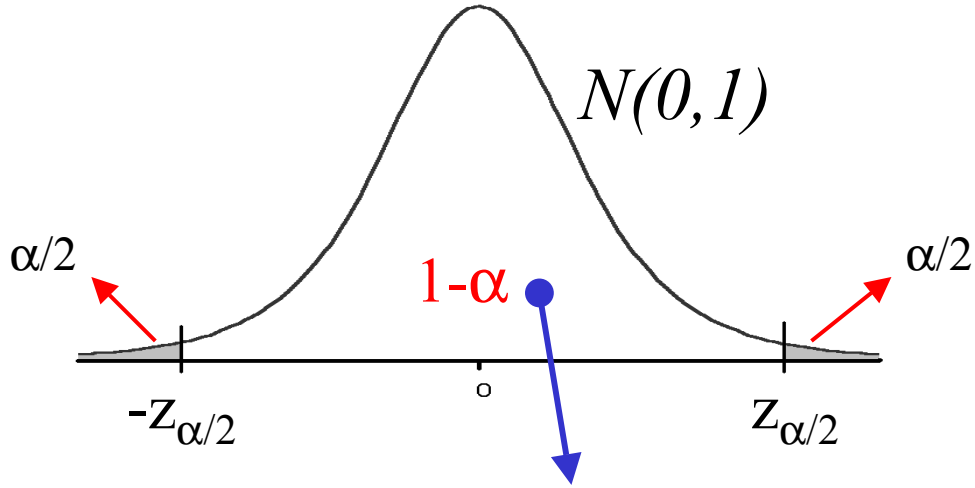
Introducción

$$X \to B(n,p) \qquad X \overset{aprox.}{\to} N(np, \sqrt{np(1-p)})$$

$$\hat{p} = \frac{X}{n} \to N(p, \sqrt{\frac{p(1-p)}{n}})$$

$$\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \to N(0,1)$$





Nivel de CONFIANZA

$$P(-z_{\alpha/2} \le \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \le z_{\alpha/2}) = 1-\alpha$$

Despejando p de:

$$-z_{\alpha/2} \le \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \le z_{\alpha/2}$$

$$-z_{\alpha/2} \leq \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \leq z_{\alpha/2}$$

$$\Downarrow$$

$$-z_{\alpha/2} \leq \frac{\hat{p}-p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} \leq z_{\alpha/2}$$

$$\Downarrow$$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq \quad p \quad \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

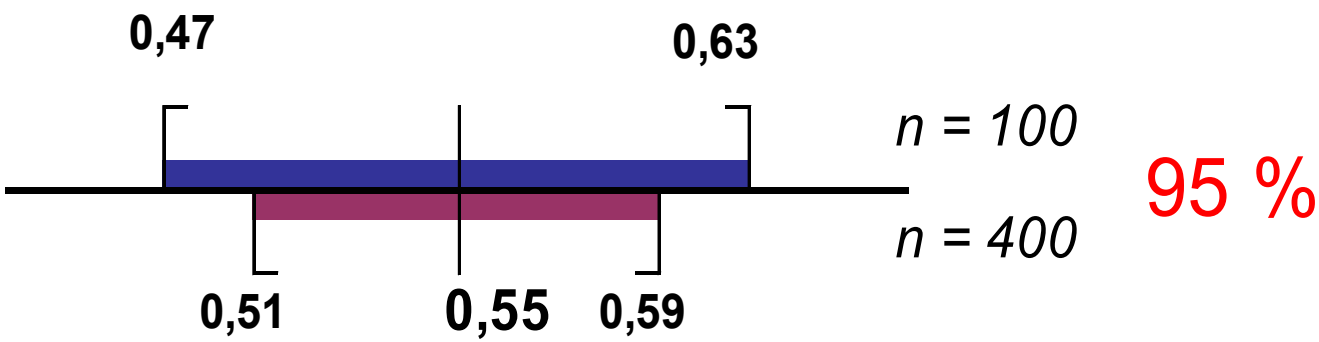
Nivel de confianza: (1-α)

Tamaño Muestral n

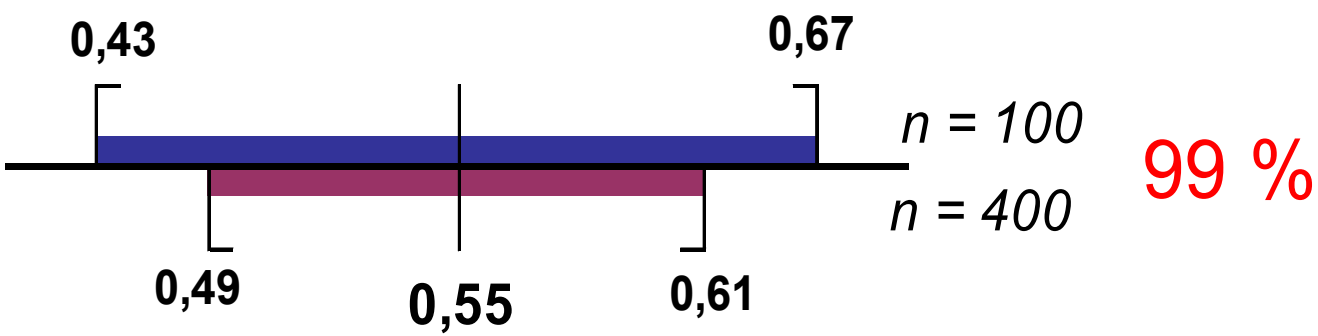
Ejemplo

$$\hat{p} = \frac{220}{400} = 0{,}55$$

$$p \in 0{,}55 \pm 1{,}96\sqrt{\frac{0{,}55 \times 0{,}45}{400}}$$



95 %



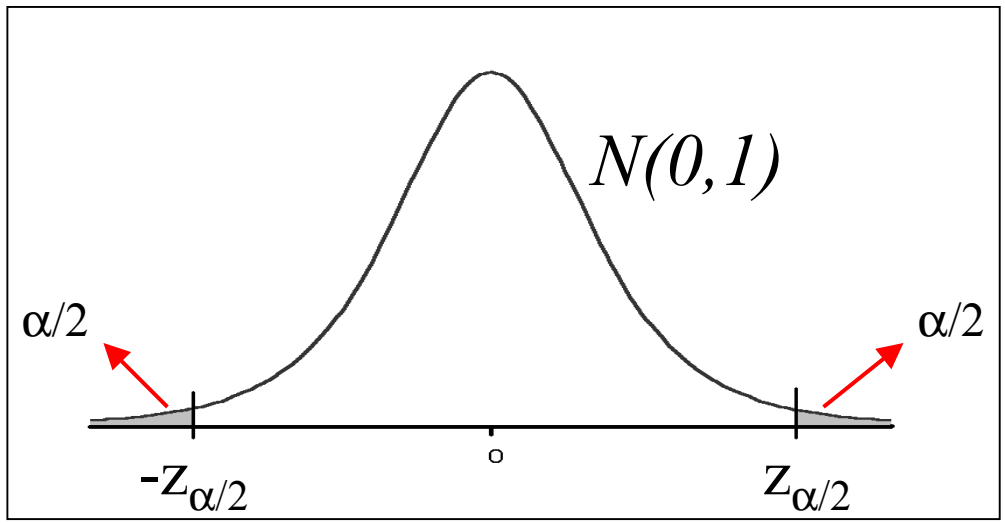
99 %

1. Normal: Intervalo para μ con σ conocido

$$X_1, X_2, \ldots, X_n \rightarrow N(\mu, \sigma)$$

$$\overline{X} \rightarrow N(\mu, \frac{\sigma}{\sqrt{n}})$$

$$\frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \rightarrow N(0,1)$$



$$-z_{\alpha/2} \leq \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \leq z_{\alpha/2}$$

$$\overline{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$\mu \in \overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

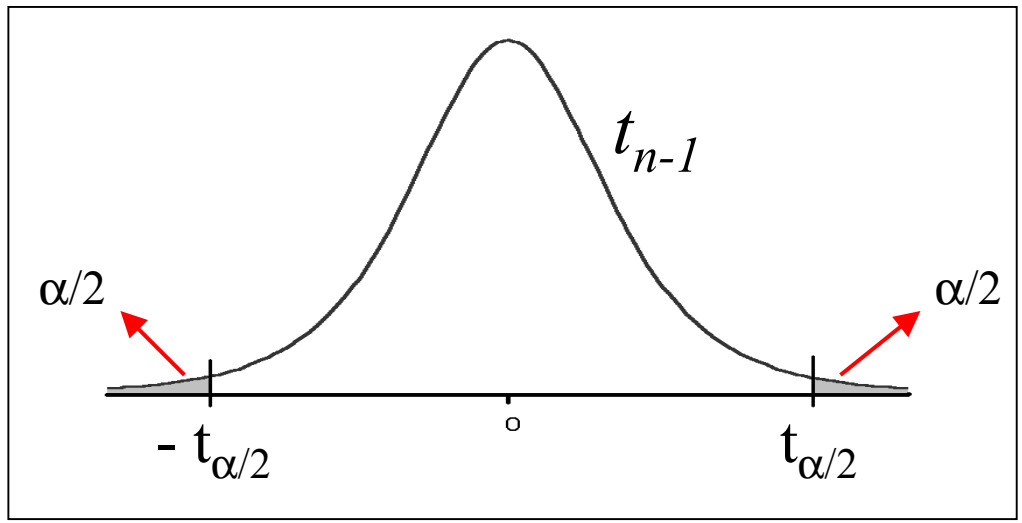
2. Normal:Intervalo para μ con σ desconocido

$$X_1, X_2, \ldots, X_n \rightarrow N(\mu, \sigma)$$

$$\overline{X} \rightarrow N(\mu, \frac{\sigma}{\sqrt{n}})$$

$$\frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \rightarrow N(0,1)$$

$$\frac{\overline{X} - \mu}{\hat{S} / \sqrt{n}} \rightarrow t_{n-1}$$



$$-t_{n-1,\alpha/2} \leq \frac{\overline{X} - \mu}{\hat{S} / \sqrt{n}} \leq t_{n-1,\alpha/2}$$

$$\overline{x} - t_{n-1,\alpha/2} \frac{\hat{s}}{\sqrt{n}} \leq \mu \leq \overline{x} + t_{n-1,\alpha/2} \frac{\hat{s}}{\sqrt{n}}$$

$$\mu \in \overline{x} \pm t_{n-1,\alpha/2} \frac{\hat{s}}{\sqrt{n}}$$

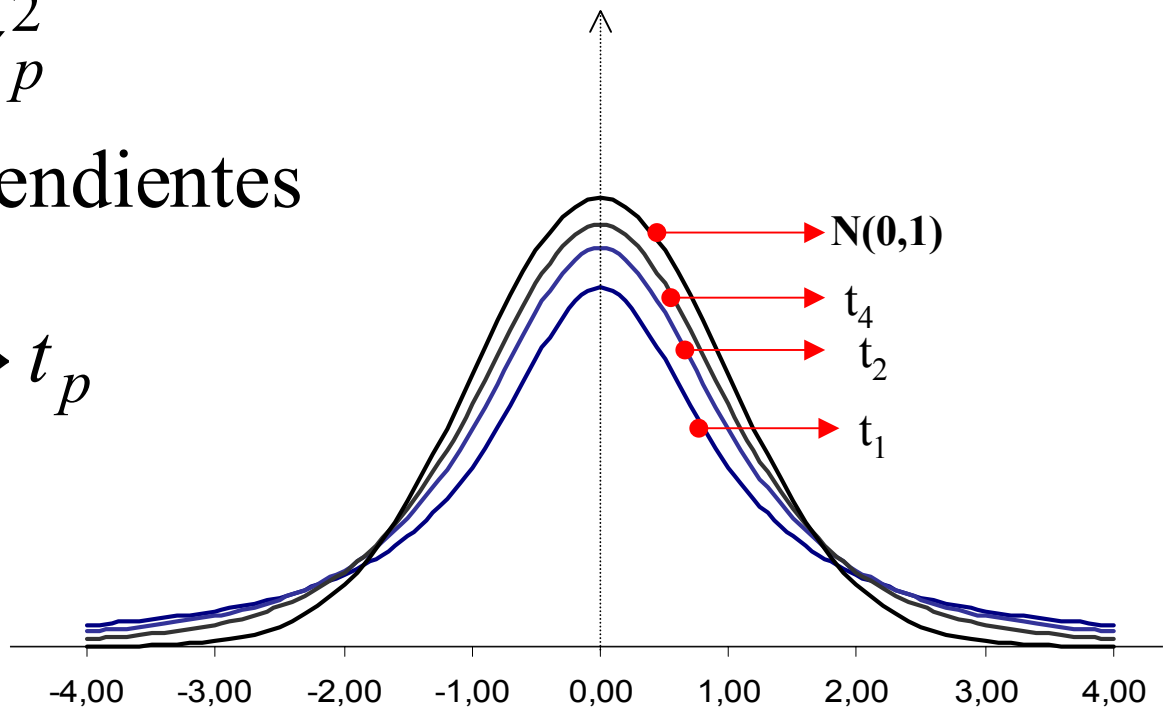
Distribución **t** de *Student*

$$Z \to N(0,1)$$

$$V \to \chi^2_p$$

Z, V son independientes

$$\frac{Z}{\sqrt{V/p}} \to t_p$$



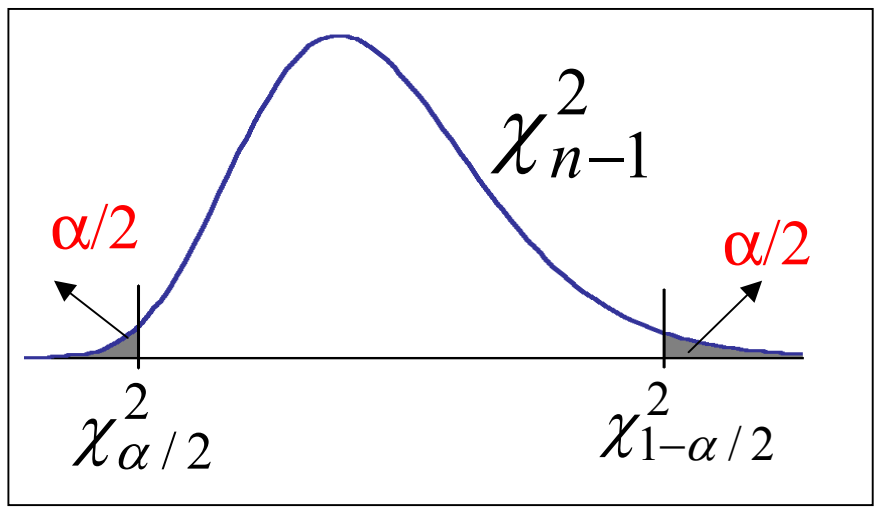
3. Normal: Intervalo para σ^2

$$X_1, X_2, \ldots, X_n \rightarrow N(\mu, \sigma)$$

$$\hat{S}^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1} \longrightarrow \frac{(n-1)\hat{S}^2}{\sigma^2} \rightarrow \chi^2_{n-1}$$

$$P\left(\chi^2_{\alpha/2} \leq \frac{(n-1)\hat{S}^2}{\sigma^2} \leq \chi^2_{1-\alpha/2}\right) = 1-\alpha$$

$$\frac{(n-1)\hat{s}^2}{\chi^2_{1-\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)\hat{s}^2}{\chi^2_{\alpha/2}}$$

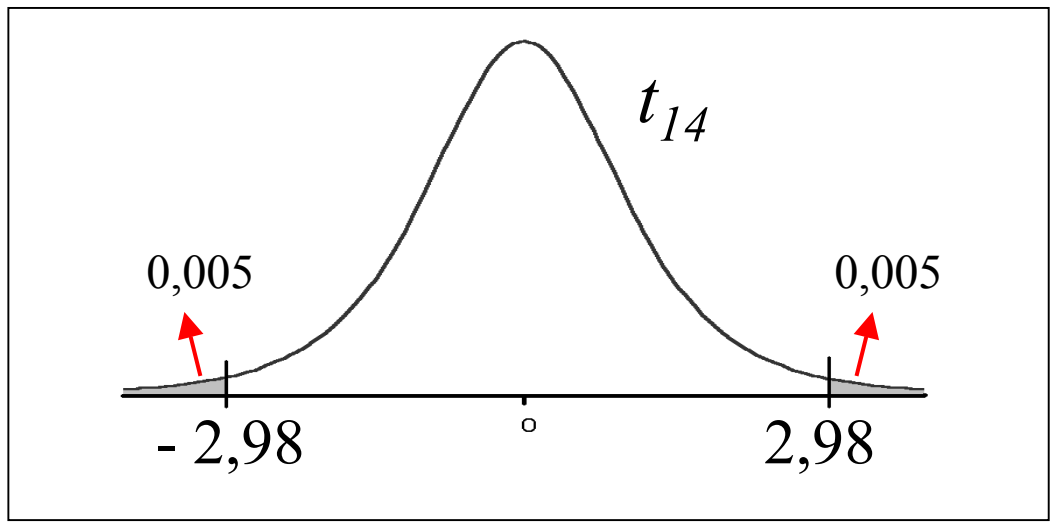


EJEMPLO 1. La resistencia a la compresión de 15 probetas de acero elegidas al azar es:

40,15	65,10	49,50	22,40	38,20
60,40	43,40	26,35	31,20	55,60
47,25	73,20	35,90	45,25	52,40

$$\frac{\overline{X}-\mu}{\hat{S}/\sqrt{15}} \rightarrow t_{14}$$

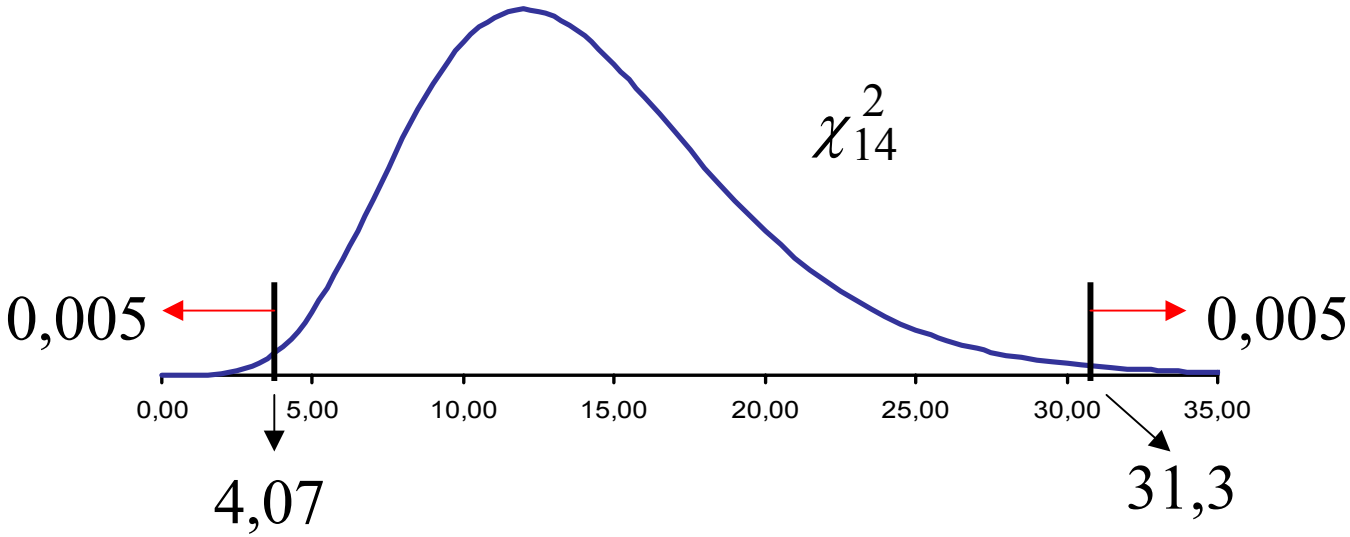
$$-2{,}98 \leq \frac{\overline{X}-\mu}{\hat{S}/\sqrt{15}} \leq 2{,}98$$



$$\overline{x} = 45{,}75 \quad \hat{s} = 14{,}2$$

$$45{,}75 - 2{,}98\frac{14{,}2}{\sqrt{15}} \leq \mu \leq 45{,}75 + 2{,}98\frac{14{,}2}{\sqrt{15}}$$

99 % confianza: $34{,}8 \leq \mu \leq 56{,}7$



$$\frac{(n-1)\hat{S}^2}{\sigma^2} \rightarrow \chi^2_{n-1} \qquad \frac{14\hat{S}^2}{\sigma^2} \rightarrow \chi^2_{14}$$

$$P(4{,}07 \leq \frac{14\hat{S}^2}{\sigma^2} \leq 31{,}3) = 0{,}99$$

$$\hat{s}^2 = 201{,}6 \longrightarrow \frac{14 \times 201{,}6}{31{,}3} \leq \sigma^2 \leq \frac{14 \times 201{,}6}{4{,}07}$$

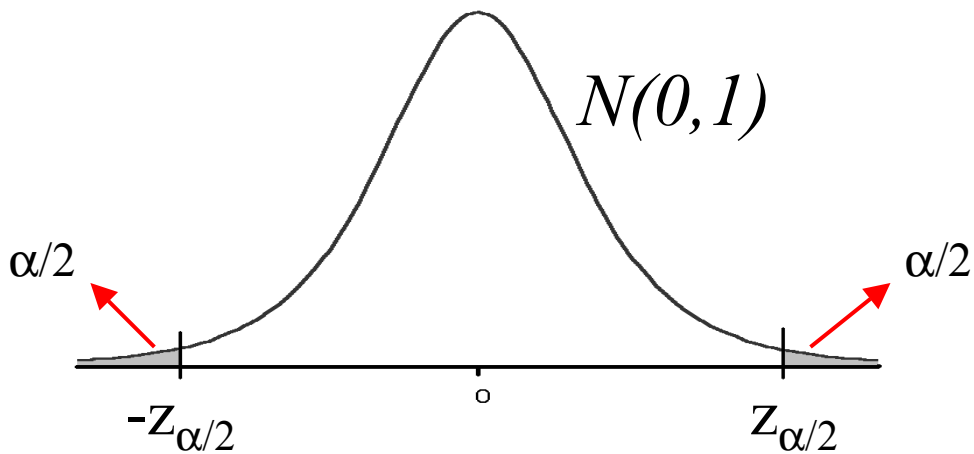
99% confianza: $90{,}2 \leq \sigma^2 \leq 693{,}6$

4. Poisson: Intervalo para λ

$$X_1, X_2, \ldots, X_n \rightarrow \text{Poisson}(\lambda)$$

$$\hat{\lambda} = \overline{X} \overset{aprox}{\rightarrow} \text{N}(\lambda, \sqrt{\frac{\lambda}{n}})$$

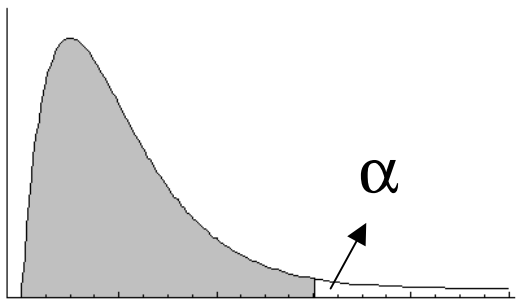
$$\frac{\hat{\lambda} - \lambda}{\sqrt{\frac{\lambda}{n}}} \rightarrow \text{N}(0,1)$$



$$P(-z_{\alpha/2} \leq \frac{\hat{\lambda} - \lambda}{\sqrt{\lambda / n}} \leq z_{\alpha/2}) = 1 - \alpha$$

$$\hat{\lambda} - z_{\alpha/2}\sqrt{\frac{\hat{\lambda}}{n}} \leq \lambda \leq \hat{\lambda} + z_{\alpha/2}\sqrt{\frac{\hat{\lambda}}{n}}$$

Tabla χ^2



$\chi_{\nu,1-\alpha}$

ν: grados de libertad (g.l.)

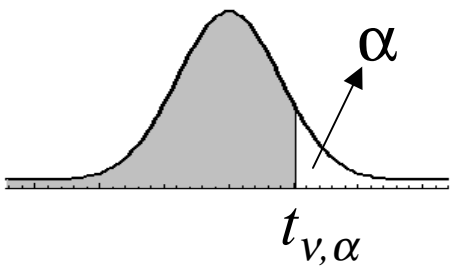
EJEMPLO

$P(\chi_9 \geq 19{,}02) = 0{,}025$

	α								
g.l.	**0,995**	**0,990**	**0,975**	**0,950**	**0,500**	**0,050**	**0,025**	**0,010**	**0,005**
1	,00004	,00016	,00098	,00393	0,455	3,841	5,024	6,635	7,879
2	,01002	,0201	0,051	0,103	1,386	5,991	7,378	9,210	10,60
3	,0717	0,115	0,216	0,352	2,366	7,815	9,348	11,34	12,84
4	0,207	0,297	0,484	0,711	3,357	9,488	11,14	13,28	14,86
5	0,412	0,554	0,831	1,145	4,351	11,07	12,83	15,09	16,75
6	0,676	0,872	1,237	1,635	5,348	12,59	14,45	16,81	18,55
7	0,989	1,239	1,690	2,167	6,346	14,07	16,01	18,48	20,28
8	1,344	1,647	2,180	2,733	7,344	15,51	17,53	20,09	21,95
9	1,735	2,088	2,700	3,325	8,343	16,92	19,02	21,67	23,59
10	2,156	2,558	3,247	3,940	9,342	18,31	20,48	23,21	25,19
11	2,603	3,053	3,816	4,575	10,341	19,68	21,92	24,73	26,76
12	3,074	3,571	4,404	5,226	11,340	21,03	23,34	26,22	28,30
13	3,565	4,107	5,009	5,892	12,340	22,36	24,74	27,69	29,82
14	4,075	4,660	5,629	6,571	13,339	23,68	26,12	29,14	31,32
15	4,601	5,229	6,262	7,261	14,339	25,00	27,49	30,58	32,80
16	5,142	5,812	6,908	7,962	15,338	26,30	28,85	32,00	34,27
17	5,697	6,408	7,564	8,672	16,338	27,59	30,19	33,41	35,72
18	6,265	7,015	8,231	9,390	17,338	28,87	31,53	34,81	37,16
19	6,844	7,633	8,907	10,117	18,338	30,14	32,85	36,19	38,58
20	7,434	8,260	9,591	10,851	19,337	31,41	34,17	37,57	40,00
21	8,034	8,897	10,283	11,591	20,337	32,67	35,48	38,93	41,40
22	8,643	9,542	10,982	12,338	21,337	33,92	36,78	40,29	42,80
23	9,260	10,196	11,689	13,091	22,337	35,17	38,08	41,64	44,18
24	9,886	10,856	12,401	13,848	23,337	36,42	39,36	42,98	45,56
25	10,520	11,524	13,120	14,611	24,337	37,65	40,65	44,31	46,93
26	11,160	12,198	13,844	15,379	25,336	38,89	41,92	45,64	48,29
27	11,808	12,878	14,573	16,151	26,336	40,11	43,19	46,96	49,65
28	12,461	13,565	15,308	16,928	27,336	41,34	44,46	48,28	50,99
29	13,121	14,256	16,047	17,708	28,336	42,56	45,72	49,59	52,34
30	13,787	14,953	16,791	18,493	29,336	43,77	46,98	50,89	53,67
40	20,707	22,164	24,433	26,509	39,335	55,76	59,34	63,69	66,77
50	27,991	29,707	32,357	34,764	49,335	67,50	71,42	76,15	79,49
60	35,534	37,485	40,482	43,188	59,335	79,08	83,30	88,38	91,95
70	43,275	45,442	48,758	51,739	69,334	90,53	95,02	100,43	104,21
80	51,172	53,540	57,153	60,391	79,334	101,88	106,63	112,33	116,32
90	59,196	61,754	65,647	69,126	89,334	113,15	118,14	124,12	128,30
100	67,328	70,065	74,222	77,929	99,334	124,34	129,56	135,81	140,17
120	83,852	86,923	91,573	95,705	119,334	146,57	152,21	158,95	163,65

Tabla

t-Student



ν: grados de libertad (g.l.)

EJEMPLO

$P(t_9 \geq 2{,}262) = 0{,}025$

	α									
g.l	**0,20**	**0,15**	**0,10**	**0,05**	**0,025**	**0,01**	**0,005**	**0,0025**	**0,001**	**0,0005**
1	1,376	1,963	3,078	6,314	12,706	31,821	63,656	127,321	318,289	636,578
2	1,061	1,386	1,886	2,920	4,303	6,965	9,925	14,089	22,328	31,600
3	0,978	1,250	1,638	2,353	3,182	4,541	5,841	7,453	10,214	12,924
4	0,941	1,190	1,533	2,132	2,776	3,747	4,604	5,598	7,173	8,610
5	0,920	1,156	1,476	2,015	2,571	3,365	4,032	4,773	5,894	6,869
6	0,906	1,134	1,440	1,943	2,447	3,143	3,707	4,317	5,208	5,959
7	0,896	1,119	1,415	1,895	2,365	2,998	3,499	4,029	4,785	5,408
8	0,889	1,108	1,397	1,860	2,306	2,896	3,355	3,833	4,501	5,041
9	0,883	1,100	1,383	1,833	2,262	2,821	3,250	3,690	4,297	4,781
10	0,879	1,093	1,372	1,812	2,228	2,764	3,169	3,581	4,144	4,587
11	0,876	1,088	1,363	1,796	2,201	2,718	3,106	3,497	4,025	4,437
12	0,873	1,083	1,356	1,782	2,179	2,681	3,055	3,428	3,930	4,318
13	0,870	1,079	1,350	1,771	2,160	2,650	3,012	3,372	3,852	4,221
14	0,868	1,076	1,345	1,761	2,145	2,624	2,977	3,326	3,787	4,140
15	0,866	1,074	1,341	1,753	2,131	2,602	2,947	3,286	3,733	4,073
16	0,865	1,071	1,337	1,746	2,120	2,583	2,921	3,252	3,686	4,015
17	0,863	1,069	1,333	1,740	2,110	2,567	2,898	3,222	3,646	3,965
18	0,862	1,067	1,330	1,734	2,101	2,552	2,878	3,197	3,610	3,922
19	0,861	1,066	1,328	1,729	2,093	2,539	2,861	3,174	3,579	3,883
20	0,860	1,064	1,325	1,725	2,086	2,528	2,845	3,153	3,552	3,850
21	0,859	1,063	1,323	1,721	2,080	2,518	2,831	3,135	3,527	3,819
22	0,858	1,061	1,321	1,717	2,074	2,508	2,819	3,119	3,505	3,792
23	0,858	1,060	1,319	1,714	2,069	2,500	2,807	3,104	3,485	3,768
24	0,857	1,059	1,318	1,711	2,064	2,492	2,797	3,091	3,467	3,745
25	0,856	1,058	1,316	1,708	2,060	2,485	2,787	3,078	3,450	3,725
26	0,856	1,058	1,315	1,706	2,056	2,479	2,779	3,067	3,435	3,707
27	0,855	1,057	1,314	1,703	2,052	2,473	2,771	3,057	3,421	3,689
28	0,855	1,056	1,313	1,701	2,048	2,467	2,763	3,047	3,408	3,674
29	0,854	1,055	1,311	1,699	2,045	2,462	2,756	3,038	3,396	3,660
30	0,854	1,055	1,310	1,697	2,042	2,457	2,750	3,030	3,385	3,646
40	0,851	1,050	1,303	1,684	2,021	2,423	2,704	2,971	3,307	3,551
50	0,849	1,047	1,299	1,676	2,009	2,403	2,678	2,937	3,261	3,496
60	0,848	1,045	1,296	1,671	2,000	2,390	2,660	2,915	3,232	3,460
70	0,847	1,044	1,294	1,667	1,994	2,381	2,648	2,899	3,211	3,435
80	0,846	1,043	1,292	1,664	1,990	2,374	2,639	2,887	3,195	3,416
90	0,846	1,042	1,291	1,662	1,987	2,368	2,632	2,878	3,183	3,402
100	0,845	1,042	1,290	1,660	1,984	2,364	2,626	2,871	3,174	3,390
infinito	0,842	1,036	1,282	1,645	1,960	2,327	2,576	2,808	3,091	3,291
	0,20	**0,15**	**0,10**	**0,05**	**0,025**	**0,01**	**0,005**	**0,0025**	**0,001**	**0,0005**